The “Big $M$” Method

Modify the LP

1. If any functional constraints have negative constants on the right side, multiply both sides by $-1$ to obtain a constraint with a positive constant.
# The “Big $M$” Method

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2. Introduce a **slack variable** $s_i \geq 0$ for each ‘$\leq$’ constraint.
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2. Introduce a **slack variable** $s_i \geq 0$ for each ‘$\leq$’ constraint.

3. Introduce a **surplus variable** $s_j \geq 0$ and an **artificial variable** $\bar{x}_i \geq 0$ for each ‘$\geq$’ constraint.
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4. Introduce an **artificial variable** $\bar{x}_j \geq 0$ in each ‘$=$’ constraint.

5. For each artificial variable $\bar{x}_i$, add a **penalty term** ‘$-M\bar{x}_i$’ to the objective function. Use the same constant $M$ for all the artificial variables. (*In numerical software, use a very large number for $M$.*)
Example (Big M in Action)

Maximize $P = 2x_1 + x_2$

subject to

\[
\begin{align*}
  x_1 + x_2 &\leq 10 \\
  -x_1 + x_2 &\geq 2
\end{align*}
\]

with $x_1, x_2 \geq 0$. 

Use Maple
The “Big $M$” Method: Example

Example (Big M in Action)

Maximize $P = 2x_1 + x_2$
subject to

\[ x_1 + x_2 \leq 10 \]
\[ -x_1 + x_2 \geq 2 \]

with $x_1, x_2 \geq 0$.

The Big $M$ Simplex Tableau

<table>
<thead>
<tr>
<th>Eq</th>
<th>Z</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$\bar{x}_1$</th>
<th>$b$</th>
</tr>
</thead>
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<tr>
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<td>10</td>
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<tr>
<td>(2)</td>
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<td>$-1$</td>
<td>1</td>
<td>0</td>
<td>$-1$</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
Exercise (O-Jay)

O-Jay is a mixture of orange juice and orange soda. We need to restrict the amount of sugar to 4gm/bottle and maintain at least 20mg/bottle of vitamin C. What is the least cost mixture?

Let:

- $x_1 = \text{number of ounces of orange soda in a bottle of O-Jay}$
- $x_2 = \text{number of ounces of orange juice in a bottle of O-Jay}$

The LP is:

Minimize $z = 2x_1 + 3x_2$

subject to

- $0.5x_1 + 0.25x_2 \leq 4$ \hspace{1cm} (sugar constraint)
- $x_1 + 3x_2 \geq 20$ \hspace{1cm} (Vitamin C constraint)
- $x_1 + x_2 = 10$ \hspace{1cm} (10 oz in per bottle)

with $x_1, x_2 \geq 0$
The “Big $M$” Method: Summary

Summary

1. If the problem is “minimize $Z$,” change to “maximize $(-Z)$.”
2. Add a slack variable $s_i$ to change ‘$\leq$’ to ‘$=$’.
3. Subtract a surplus variable $s_j$ and add an artificial variable $\bar{x}_j$ to change ‘$\geq$’ to ‘$=$’.
4. Add an artificial variable $\bar{x}_k$ to each ‘$=$’ constraint.
5. Add ‘$-M \bar{x}_j$’ to the objective function for each artificial variable $\bar{x}_j$.
6. Use a row operation with each artificial variable row to eliminate $M$ from the objective function in $\bar{x}_j$ columns.
7. Run the simplex algorithm.
The “Big M” Method: Summary

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## Summary

1. If the problem is “minimize $Z$,” change to “maximize ($-Z$).”

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6. Use a row operation with each artificial variable row to eliminate $M$ from the objective function in $\bar{x}_j$ columns.\(^1\)

---

\(^1\) Just operate on Row (0), the objective function, to reduce arithmetic.
The “Big $M$” Method: Summary

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1. If the problem is “minimize $Z$,” change to “maximize $(-Z)$.”

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7. Run the simplex algorithm.

¹Just operate on Row (0), the objective function, to reduce arithmetic.
The “Big $M$” Method: Big Example

Example (Big “Big M”)

Maximize

\[ Z = 2x_1 + 5x_2 + 3x_3 \]

subject to

\[
\begin{align*}
    x_1 + 2x_2 - x_3 & \leq 7 \\
    -x_1 + x_2 - 2x_3 & \leq -5 \\
    x_1 + 4x_2 + 3x_3 & \geq 1 \\
    2x_1 - x_2 + 4x_3 & = 6
\end{align*}
\]

with \( x_1, x_2, x_3 \geq 0 \).
The “Big $M$” Method: Big Example

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with \quad x_1, x_2, x_3 \geq 0.

Variables

There are:

• 3 decision variables \( x_i \)
• 1 slack variable \( s_1 \)
• 2 surplus variables \( s_j \)
• 3 artificial variables \( \bar{x}_j \)
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Initial Artificial Problem Tableau

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<tr>
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<th>$x_3$</th>
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### Beginning Simplex Tableau

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