Work quickly and carefully, following directions closely. Answer all questions completely.

FOR ALL PROBLEMS: Define $P$, $Q$, $R$, and $S$ to be the four digits in your given number.

\[ P = \_, \quad Q = \_, \quad R = \_, \quad S = \_ . \]

§I. TRUE and/or FALSE. Circle your answer. There are 2 questions at 2 points each.

1. TRUE or FALSE: Every function has a Taylor series at every point.
2. TRUE or FALSE: The absolute error divided by the target value equals the relative error.

§II. MULTIPLE CHOICE. Circle your answer. There are 2 question at 5 points each.

1. The Mean Value theorem is based on
   (a) Extreme Value theorem  \hspace{1em} (b) Intermediate Value theorem  \hspace{1em} (c) the integral of constant sign function is nonzero
   \hspace{1em} (d) none of the above  \hspace{1em} (e) all of the above

2. The Discrete Average Value theorem says
   (a) discrete values are averaged by summing, then dividing by the number of values
   (b) a function takes on any ‘percentage combination’ of its values if the percentages total to 100%
   (c) the integral of constant sign function is nonzero
   (d) none of the above
   (e) all of the above

§III. PROBLEMS. You must show your work to receive credit. There are 4 problems at 20 points each.

1. Let $f(x) = \tan(\sin(x))$.
   (a) Write the Taylor polynomial $T_1$ for $f$ centered at $c = 0$ to $O(x^2)$.
   (b) Determine the maximum error for $T_1$ approximating $f$ for $x \in [-1, 1]$.

2. Use Taylor’s theorem to show that $\frac{\sin(x)}{x} = 1 + O(x^2)$.

3. Show that
   \[ f''(x) = \frac{f(x + h) - 2f(x) + f(x - h)}{h^2} + O(h^2) \]
   for all $h$ near 0.

4. Explain why $(0.5 - 0.4 - 0.1) \neq 0$ in IEEE-754 arithmetic.