Women and Minorities in Mathematics

Incorporating Their Mathematical Achievements Into School Classrooms

Marjorie Lee Browne: North Carolina Educator

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Marjorie Lee Browne was a pioneering woman in mathematics. She was one of the earliest black women to receive a PhD in mathematics and she was well known as an extremely caring and effective North Carolina educator.

Early Black Women Mathematicians

The earliest black women in mathematics faced many barriers:

> *Over the years, black women who might be disposed to pursue a career in mathematics faced the "double whammy" of racism and sexism. Like blacks, women were not considered to have the mental skills necessary for advanced mathematical inquiry. For all women, and especially for black women, the field of mathematics was essentially shut tight.* (The Journal of Blacks in Higher Education, 2001)

Things began to change as black women obtained PhDs in mathematics, making it easier for others who followed, but there is still much work to be done. Today, less than 1% of all mathematicians are black, and of those, approximately 25% are women (Williams, 2005).

Until recently, it was thought that Evelyn Boyd Granville, who received her PhD in 1949 from Yale University in functional analysis, and Marjorie Lee Browne, who finished her requirements for graduation in 1949, but received her PhD in 1950 from the University of Michigan in topological and matrix groups, were the first black women PhDs in mathematics. Scott Williams, who created and maintains the *Mathematicians of the African Diaspora* website (Williams, 2005) says:

> *Until 2001, everyone thought one person, Boyd-Granville, was the first African American woman to earn a Ph.D. (1949), when we discovered another to have done it in 1943. This means that still, all books published on the subject are incorrect.* (Williams, 2004)

When Williams was informed of the existence of an earlier PhD, he created a web page profile, and it is through this profile that the correction is being publicized: Martha Euphemia Lofton Haynes received her PhD in 1943 from Catholic University of America by writing a thesis on the *Determination of Sets of Independent Conditions Characterizing Certain Special Cases of Symmetric Correspondences*. Marjorie Lee Browne completed her degree seven years later.

Background and Education

Marjorie was born on September 9, 1914 in Memphis, Tennessee. She comes from a supportive family with mathematical talent.

While it was unusual at the time due to racial barriers, her father had been able to attend two years of college and was known for being a "whiz" in mental arithmetic. He helped both Marjorie and her older brother in their mathematical studies, both financially and by passing on his love for mathematics. While her brother majored in mathematics as an undergraduate, he obtained a master's degree in physical education and pursued that as a career.

It is not surprising that Marjorie attended and taught at historically black institutions, since there were very few opportunities for blacks at other learning institutions. She attended an excellent private black high school in Memphis. Even though college funding was difficult during the depression, with the help of loans, scholarships, and hard work, she graduated from Howard University, a historically black institution. Next she taught at a black high school in New Orleans. When a neighbor informed her that the University of Michigan was open to blacks and within her price range, she attended during the summers to obtain her master's degree. She then taught at Wiley College in Texas, a historically black institution, as she worked on her PhD at Michigan. Her thesis advisor was George Rainich, who had encouraged and advised numerous black students, including Wade Ellis Sr. Her PhD thesis was on *Studies of One Parameter Subgroups of Certain Topological and Matrix Groups.* Although Browne applied to research universities, she was politely rejected:

> *Browne was acutely aware of the obstacles which women and minorities faced in pursuing scientific careers... and resolved that her greatest contributions would be directing programs designed to strengthen the mathematical preparation of secondary school teachers and to increase the presence of minorities and females in mathematical science careers.* (Fletcher, 1999)

She accepted a position at the historically black institution North Carolina College, which is now North Carolina Central University.

North Carolina Central University

Marjorie poured her heart into her job:

> *If I had my life to live again, I wouldn't do anything else. I love mathematics*. (Kenschaft, 1980)

As a mathematics professor, Marjorie taught fifteen hours a week, published articles, ran workshops for secondary teachers in the summers, and became the chair of the mathematics department. Browne described herself a "pre-*Sputnik* mathematician," referring to pure mathematics preparation and research, done for the sake of intellectual pursuit, instead of applied investigations (Fletcher, 1999). When she was asked for a definition of this term, she said that it was

> *a mathematician who appreciates the beauty, power and eloquence of mathematics as one of the greatest art forms; one who responds to its ability to stir the imagination and who recognizes mathematics as the sole custodian of precision.* (Fletcher, 2005)

She was an outstanding teacher and she was the advisor of ten master's theses. Dr. Asamoah Nkwanta, who was a student at Central and met her shortly before her retirement, said:

> *She was a very nice and down-to-Earth type of person. She always encouraged us as students. I was and have always been impressed with her mathematical endeavors. I was honored to have met her.* (Nkwanta, 2005)

Dr. Fletcher, who first met her as a student in her calculus class in 1954, said:

> *It was Dr. Browne who first showed me and countless other bewildered students that mathematics could be a delightful, creative pursuit; and her forbearance and encouragement made life for us - as beginning mathematics students - challenging and brighter. She brought mathematics within our reach and opened up a world of beauty and opportunities that we did not know existed... Her philanthropic philosophy included the axiom that no good student should go without an education simply because he or she lacked the financial resources to pay for it. Thus, it was not uncommon for her to provide financial assistance for many of her students - - for tuition, books, food, clothes, funds to attend scientific meetings.* (Fletcher, 2005).

At the national level, she advocated for the integration of mathematics meetings and conferences and she was an outspoken critic of discrimination that was prevalent in funding agencies (Fletcher, 1999). She received numerous grants, honors and awards, including the first W. W. Rankin Memorial Award for excellence in mathematics education from the North Carolina Council of Teachers of Mathematics. She died in 1979.

Activities and NCTM Standards

The following activities relate to her mathematics and address numerous points in the NCTM *Principles and Standards for School Mathematics*. While the inclusion of her mathematical achievements can easily be incorporated into college level modern algebra courses, aspects relating to her work may still be incorporated into other levels.

In grades three through five, students explore how variables are related to each other, and the one-parameter activity can be adapted for use there or used in the higher grades to explore the connections between algebra and geometry. As in the worksheet presented below on matrix groups, which is designed for use in the high school classroom, the number and operations standard specifies that students should develop their understanding of properties and representations of addition and multiplication of matrices. The standards also discuss the fact that "mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement." (National Council of Teachers of Mathematics, 2000) The North Carolina Mathematicians activity can help with this goal.

One-Parameter Activity

Begin with an introduction to Marjorie Lee Browne's life and explain that her thesis related to one-parameter objects. Have your students search the internet for the definition of a parameter. Ask them to create a function of one variable, to write the (x,y) coordinates of a circle in terms of one variable, and to compare their work with the definition of parameter they found. Ask them to share their findings with the rest of the class and discuss the difference between a variable and a parameter. Search the internet for applications of one-parameter objects within other fields such as statistics and the mathematics of diseases.

Activity Sheet: Matrix Groups

The following activity sheet gives high school students a chance to encounter some of the matrix groups and techniques from Marjorie's thesis and published paper and to explore the connections between geometry, algebra, and linear algebra. While a college level student could explore more of her work, this worksheet is aimed at high school students. Solutions can be found at http://www.mathsci.appstate.edu/centroid/.

North Carolina Mathematicians Activity

Have student groups research the lives and work of Marjorie Lee Browne and other prominent mathematicians who taught in North Carolina such as Alfred Brauer (UNC Chapel Hill and Wake Forest University), Ethelbert Chukwu (North Carolina State University), William Cochran (North Carolina Institute of Statistics), Elbert Frank Cox (Shaw University), Gertrude Cox (North Carolina State University and North Carolina Institute of Statistics), Max Dehn (Black Mountain College), William Fletcher (North Carolina Central University), Johnny Houston (Elizabeth City State University), Witold Hurewicz (UNC Chapel Hill), Mary Ellen Rudin (Duke University), Beauregard Stubblefield (Appalachian State University), and William Whyburn (UNC Chapel Hill). These are just a sampling of prominent North Carolina mathematicians. By using a google search ("North Carolina" site: www-groups.dcs.st-and.ac.uk), and other similar site searches (e.g., The Mathematics Genealogy Project; Williams, 2005), many additional names can be found.

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Activity Sheet: Matrix Groups

Marjorie Lee Browne was the third black woman to receive her PhD in mathematics in the United States and she was known as an extremely caring and effective North Carolina educator, teaching at North Carolina Central University for most of her career. With regards to her career choice, Dr. Browne said: "If I had my life to live again, I wouldn't do anything else. I love mathematics." In this worksheet we will explore topics related to her thesis work and published paper.

A *matrix group* is a set of matrices that satisfy algebraic relationships. One of the many groups of matrices studied by Dr. Browne was the group of square orthogonal matrices, represented by the notation *O(n)*, where *n* represents the dimension of the matrices. In order to understand what it means to be an orthogonal matrix, we must first investigate identities and transposes.

We learn in elementary school that the number 1 multiplied by any other number, always results in that number. 1 is called the multiplicative identity because of this fact. Sets of square matrices also have multiplicative identities, called *identity matrices*. An identity matrix ***I*** is a matrix with 1's down the diagonal and 0's everywhere else. For any n×n matrix *A*, the product of *A* with the *n*×*n* identity matrix ***I***, is *A*.

<u>Question A</u> Check the identity property with, $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ and $\boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

<u>Question B</u> Is multiplication with ***I*** commutative ($A\boldsymbol{I}=\boldsymbol{I}A$)?

The *transpose* of a matrix is a new matrix whose rows are the columns of the first matrix and whose columns are the rows. For example, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ has transpose $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$. A square matrix is *orthogonal* if the product of itself with its transpose is the identity matrix, so that $AA^T=\boldsymbol{I}$.

<u>Question C</u> Are $B = \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ and $C = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ -2 & 0 \end{bmatrix}$ orthogonal matrices?

In terms of geometry, the symmetries of the circle $x^2 + y^2 = 1$ are *O(2)*, the symmetries of the sphere $x^2 + y^2 + z^2 = 1$ are *O(3)*, and in general *O(n)* is extremely important in science and engineering. For example, the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ acts on the circle by matrix multiplication $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$, and so a point on the circle is sent to the corresponding point on the circle with the opposite x-value, but same y-value, i.e. across the reflection in the y-axis. Since this reflection preserves the circle, it is a symmetry and is an element of *O(2)*.

<u>Question D</u> Show that the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is orthogonal by checking that $AA^T = \boldsymbol{I}$. Next explain why it preserves the circle by describing how it acts on points on the circle as a rotation of a certain degree.

Marjorie Lee Browne also studied the *decompositions* of matrix groups. A decomposition of a matrix group is a breakdown of the group into a product of factors, so that any member of the group could be written as a product of such factors. When working with matrix groups it may be helpful to work with a group's decomposition, in order to better understand the structure of the matrices within that group. This is an example of a general philosophy in mathematics to break down a large problem into smaller or simpler problems in order to find a solution. A common decomposition is that of the real plane. Letting $\boldsymbol{R}$ represent the set of real numbers, $\boldsymbol{R}\times\boldsymbol{R}$ is the standard decomposition of the real plane, $\boldsymbol{R}^2$. We use this all the time when we specify the position of points in the plane as the decomposition of a horizontal distance from the origin and a vertical distance from the origin. Dr. Browne worked with the decompositions of many different matrix groups. Three such groups are the group of square orthogonal matrices mentioned earlier as *O(n)*, $GL(n,\boldsymbol{C})$ the matrix group of invertible square matrices of complex numbers, and $GL(n, \boldsymbol{R})$ the subgroup of $GL(n,\boldsymbol{C})$ made up of matrices with real coefficients and non-zero determinants.

<u>Question E</u> Give some examples of other types of decompositions you have utilized.

Dr. Browne also explored the notion of *connectedness*. A set is disconnected if you can break it up into two disjoint open sets and it is connected if you cannot do so.

<u>Question F</u> Create examples of both an equation and a graph of a connected set and a disconnected set.

The notion of connectedness also applies to collections of matrices. In her paper on classical matrix groups, Dr. Browne stated that:

> *It is well known that many of the underlying spaces are connected. But by proving SU(n) is connected by first principles, the connectivity of the others comes easily through a consideration of their decompositions and an elementary property of projections.*

SU(n) is the special unitary matrix group and *SU(2)* is a representation of the 3-sphere, a higher dimensional sphere denoted by $x^2 + y^2 + z^2 + w^2 = 1$. Marjorie uses *SU(n)* along with decomposition methods to help show that other groups are connected. Review the definition of connectedness. It is sometimes difficult to show that a set is connected, because it can be hard to show that it is impossible to break it up, especially in cases where visualization techniques cannot be used. Marjorie was able to break some of her matrix groups into simpler components, which she already knew were connected, and in this way show that the matrix group was connected.

<u>Question G</u> Explain how can you use the standard decomposition of the plane along with the connectivity of the real numbers in order to show that the plane is connected.

Marjorie Lee Browne's PhD thesis was on the *Studies of One Parameter Subgroups of Certain Topological and Matrix Groups*. While we will look at a few examples, the actual definition of a *one-parameter subgroup* of a matrix group (the image of a continuous group homomorphism from **R**) is beyond the scope of this worksheet. The matrix $\begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix}$ is a one-parameter subgroup of *SL(2)*, the matrix group of *2*×*2* matrices with real entries and determinant one. The parameter here is *t*, a real number.

Question H Recall that the determinant of a *2*×*2* matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is *ad-bc*. Check that $\begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix}$ has determinant 1.

The matrix *R(t)* $= \begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a one-parameter subgroup of *O(3)*, the orthogonal group mentioned at the beginning of this worksheet that act as symmetries of the sphere $x^2 + y^2 + z^2 = 1$. Review the section of the worksheet on symmetries now, as it will be helpful in answering the following questions.

Question I To show that *R(t)* is a symmetry of the sphere, calculate $\begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and show that the new values of *x, y* and *z* satisfy $x^2 + y^2 + z^2 = 1$.

Question J For a given real number *t*, what geometric transformation does the matrix *R(t)* represent? First try some examples of specific values of *t* such as $t = 0°$, $t = 90°$, and $t = 180°$ and examine the action of *R(t)* on the sphere $x^2 + y^2 + z^2 = 1$.

While her thesis contained completely original work, her published paper represented an attempt to help people understand matrix groups. In Marjorie Lee Browne's own words:

> *The purpose of this paper is to set forth some topological properties of and relations between certain classical groups. While much of the material included here may be known to a few, the main interest of this paper lies in the simplicity of the proofs of some important, though obscured, results.*

Throughout her career, she tried to make mathematics accessible to as many people as possible, both in her research and her teaching. She received numerous grants, honors and awards, including the first W. W. Rankin Memorial Award for excellence in mathematics education from the North Carolina Council of Teachers of Mathematics.

Solutions for the Marjorie Lee Browne Activity Sheet on Matrix Groups

Question A Yes $A\boldsymbol{I} = \boldsymbol{I}A = A$.

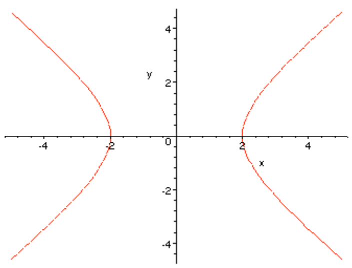
Question B Multiplication by the identity matrix is always commutative.

Question C Notice that $BB^T = \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} = \boldsymbol{I}$ and so B is an orthogonal matrix (you may note $B=B^T$ in this case), but $CC^T = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} \sqrt{3}/2 & -2 \\ -1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & 4 \end{bmatrix}$ and so C is not orthogonal.

Question D $AA^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \boldsymbol{I}$. Since $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$, A is a clockwise rotation by 90 degrees.

Question E Answers will vary. A few examples are when thinking of volume as base area multiplied times height ($V=B\times h$) or when the difference of two squares may be factored into two binomials $x^2-a^2=(x-a)(x+a)$.

Question F Many possible answers. The real numbers or the unit sphere $x^2+y^2+z^2=1$ are connected, while the hyperbola $x^2-y^2=4$ which intersects the x-axis at (2,0) and (–2,0) is disconnected.



Question G Since $\boldsymbol{R}^2 = \boldsymbol{R}\times\boldsymbol{R}$, and $\boldsymbol{R}$ is connected, so is the plane.

Question H The determinant is $e^t\ e^{-t}$-0 = e^{t-t}= e^0=1.

Question I $\begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(t)x-\sin(t)y+0z \\ \sin(t)x+\cos(t)y+0z \\ z \end{bmatrix}$. To check that this satisfies the relationship $x^2+y^2+z^2=1$, we use algebra and the trigonometric identity $\cos^2(t)+\sin^2(t)=1$, and notice that

$$[\cos(t)x-\sin(t)y]^2+[\sin(t)x+\cos(t)y]^2+z^2$$

$$=\cos^2(t)x^2-2\cos(t)\sin(t)xy+\sin^2(t)y^2+\sin^2(t)x^2+2\cos(t)\sin(t)xy+\cos^2(t)y^2+z^2$$

$$=(\cos^2(t)+\sin^2(t))x^2+(\cos^2(t)+\sin^2(t))y^2+z^2=x^2+y^2+z^2$$

Question J $R(t)$ fixes the z coordinate and rotates the x and y coordinates by t degrees clockwise in any constant z plane ($z=c$). Hence this is a rotation of the sphere about the z-axis through the north and south poles.