Incorporating the Mathematical Achievements of Women and Minority Mathematicians into Classrooms

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Introduction

There are many references for activities that incorporate general multiculturalism into the classroom (e.g., [4, 21, 29]). There are also numerous "women in mathematics" courses that focus on history and equity issues. Yet, except for a few sources such as [19], [23], [24], [26], and [27], sources that discuss women and minorities in mathematics do not include related activities for the classroom that contain significant mathematical content, and those that do are mainly aimed at the middle grades or high school level. This is unfortunate since students benefit from the inclusion of the achievements of women and minorities in mathematics classes, as "the result is that students will see mathematics as a discipline that transcends culture, time, and gender, and as a discipline for everyone, everywhere." [19, page xi]

Since there were only a handful of known women and minority mathematicians before the last 200 years, an effective study of them must focus on recent history. Projects that include such recent history are harder to create because of scarce resources for the classroom, but they are beneficial because students more readily identify with these mathematicians for the reasons stated in the next section. After examining the importance of incorporating the achievements of women and minority mathematicians into classrooms, we will discuss the methodology of historical projects about mathematicians and their mathematics. Then we will explore the implementation of these projects and the inclusion of some women and minority mathematicians as we give examples related to three living mathematicians: Andrew Wiles, Carolyn Gordon, and David Blackwell.

Importance of Incorporating Women's and Minorities' Achievements into Classrooms

Many students never learn about women and minority mathematicians although research shows that this is beneficial to all students, including white males [5, page 248]. Since interviewers often ask women and minority mathematicians specific questions related to the common themes which emerge from their lives and work, and these themes are particularly engaging for students, the availability of this information and the openness of these mathematicians about their personal struggles allows students to readily identify with them.

Common Themes

1) While subtle gender or racial discrimination can be as devastating to the career of a mathematician as overt discrimination, the support of family, teachers or mentors (male or female)

at any stage of the educational process can help overcome these difficulties.

2) The number of women and minority mathematicians has greatly increased in recent years, probably at least in part due to increased educational opportunities.

3) Most early women mathematicians did not marry. More recently, many successful women mathematicians have balanced a career and family.

4) Successful mathematicians have diverse styles, and many of them struggle with mathematics in much the same way students do.

When we included the achievements of women and minority mathematicians into an introductory class for non-majors, one student sent an unsolicited e-mail message stating: "I think it is wonderful that you are spending time in class discussing the discrepancies between men and women in the world of math and sciences. I think it is very informative and very important so thanks!"

We will now discuss the importance of including information related to the above themes in the classroom, and the reasons why women and minority mathematicians are particularly well suited for the introduction of these themes.

Importance of Including Mentoring Issues

Students relate personally to mentoring issues, and including them encourages the students to reflect on their own mathematical experiences, which is especially useful for math-phobic students and future teachers. As one student commented in evaluations: "The stereotypes that some women encountered as a woman practicing mathematical sciences are quite real to me. My father never encouraged me to succeed in math; it was always okay if I did not do my best in math. He felt that math was a subject that men naturally excelled in, leaving women to literature and sentence structure."

While white male mathematicians have also benefited from mentoring experiences, they are less likely to discuss this fact in interviews, because interviewers do not usually ask them questions related to mentoring. However, interviewers do generally ask women and minorities questions related to the support of teachers, family and mentors, and so including women and minorities in the classroom is a natural way to introduce the importance of mentoring in mathematics.

Importance of Including Living Role Models

Including some living mathematicians and women and minority mathematicians in historical projects allows students to examine issues related to the historical progression of mathematicians, such as the number of women and minority mathematicians and the ability of women mathematicians to balance a successful career and family life. In addition, our students identify more easily with living mathematicians and, in the case of women students, women mathematicians, so it is important to include them. Over the course of two years, five introductory classes were asked which mathematician they most identified with. While more than half of the mathematicians we studied were no longer living, a higher percentage of students in every class identified with a living mathematician. In addition, even though we studied an equal number of white female and white male mathematicians, along with several minority male mathematicians, in each class a disproportionately higher number of women students identified with women mathematicians. These facts are not surprising since people often identify with those who share common experiences and characteristics.

Importance of Including Mathematicians with Diverse Styles

Exploring the diverse styles of mathematicians in the classroom is especially rewarding for a number of reasons. One student reflected, "Learning about mathematicians and their styles helps you appreciate all the hard work they did to make your life easier." Not only do students appreciate and identify with these mathematicians, but including mathematicians who have different mathematical styles encourages students to reflect on how they do mathematics and helps to improve their self-confidence. As one student commented, "I have also never studied mathematicians before. Studying them made me realize that even they struggle with math. This made me feel better about my own struggles." In addition, numerous students have told us that studying the diverse ways mathematicians conduct research helped them to begin thinking of themselves as mathematicians.

While there are white male mathematicians who have discussed their mathematical styles and struggles in interviews, in-depth information related to the way they do mathematics is more often found in interviews about women and minority mathematicians.

Importance of Including Mathematics

Students relate to the above themes which arise naturally from the lives and work of women and minority mathematicians, and so including some of these mathematicians into classrooms allows students to identify with them, which in turn helps them in their own studies. However it is a disservice if we reduce these mathematicians only to their gender or race. As Julia Bowman Robinson once said "What I really am is a mathematician. Rather than being remembered as the first woman this or that, I would prefer to be remembered as a mathematician should, simply for the theorems I have proved and the problems I have solved." [7, page 271] Women and minority mathematicians should be discussed in the context of their mathematics, as advocated by Robinson. Not only does this help students view these women and minorities as real mathematicians, but it also helps students relate to the mathematics. As one student noted,

> Seeing the individuals in the mathematician section gave me a glimpse of the people in math. This was the most interesting to me. It showed the personal side of math. I think people see math as the most impersonal and generic of the subjects. This is far from the case. By seeing the work these people do I have a new respect for the math that makes much of life possible. If all people were to see these examples and applications, they would think the same.

Historical Project Methodology

When we first incorporated historical projects about an individual's life story and mathematics into classrooms, students created presentations and in some classes they also wrote a paper. While students found their projects interesting and rewarding, they were passive listeners during other presentations and did not effectively learn the related mathematics, even though they were responsible for the material on quizzes and tests. We then incorporated a classroom worksheet component, and this aspect has been extremely successful at engaging the class. Students find it rewarding to create a classroom activity sheet that is designed to engage the class with material they have presented. This type of assignment benefits the entire class and is especially appropriate for courses aimed at future teachers.

A worksheet assignment is also appropriate for courses that have been designated as "writing intensive" by the university. Students end up writing a significant amount because part of the assignment is for students to include information about their mathematician as related to the segment themes. In addition, this gives students a chance to encounter issues related to teaching and grading in a creative format. Since they create the worksheets themselves, and then hand it out to the rest of the class, they develop an ownership of the material and a pride in their writing not usually seen in writing designated courses. In some courses, we have replaced the paper component of the historical project with the worksheet assignment in order to ensure that the workload of the students does not increase.

Since these are meant as long term projects, students need to begin the projects early in order to have time to master the material. If they do not allow for this, they may present material they do not understand. Some students who have worked hard in order to understand the mathematics may present the material too quickly. In order to ensure an effective learning environment, we repeatedly expose the students to the mathematics. The class completes pre-readings on the mathematics so that they will have been exposed to it before the presentations. We also review the material after each presentation in order to correct any historical or mathematical inaccuracies and reinforce the material. Students complete the classroom worksheets and are also responsible for the material on tests.

Including Women and Minorities in Historical Projects

Historical projects that incorporate the achievements of women and minority mathematicians can be implemented in a variety of levels of college courses. It is easy to incorporate women and minorities into historical projects because of the abundance of interviews that have been published recently. Because our students easily relate to the information contained within these interviews, these projects work extremely well. It is sometimes more challenging to incorporate the related mathematics because of a lack of classroom resources designed to do so. Here we will discuss how to incorporate these projects into several different kinds of classes.

Projects in an *Introduction to Mathematics* Course

In an introductory class for non-majors, we created a three-week segment called *What is a Mathematician?* that is designed to expose students to a survey of topics in mathematics, the mathematicians who worked on these topics and the historical context. We assigned each group of two students a mathematician to be studied closely. If there were an odd number of people in the class, we asked for a volunteer to work alone. We chose the mathematicians in order to expose the students to specific mathematical content and a variety of research styles. We selected a range of mathematicians from the 1700s until today. We included women and minority mathematicians for the reasons outlined in the first section; it was easy to find women and minority mathematicians whose mathematics related to the course goals.

Because this assignment required a significant amount of time outside of class, we gave the students the resources [15] they would need in order to complete their projects so that they would not also have to search out useful references. In addition to reducing the workload, this ensured that the students used quality references.

Since this was an introductory class, we did not expect the students would have the mathematical maturity needed to determine the depth of mathematics appropriate for the class. Hence, we created a specific list of mathematics questions that related to each mathematician [13]. We asked the students to answer these questions and also asked the groups to include information related to issues of diversity, influences, support, barriers, and the mathematical style of their mathematician.

Students prepared Microsoft PowerPoint presentations and classroom worksheets in order to engage the rest of the class with mathematics related to their mathematicians. Students learned the material themselves in order to teach it to the rest of the class.

We gave the students presentation and worksheet checklists [14, 17] so that they had guidelines to follow. Since this was a teaching assignment, students graded the worksheets that they had created after the students completed them. We gave them suggestions for improvement which included suggestions from other students. Because this course is designated "writing intensive," the students were expected to revise and improve the worksheet they designed.

As the class learned about the mathematics and the mathematicians from the student presentations, we highlighted the validity and success of diverse styles and the changing role of women and minorities over time.

Projects in a *Modern Algebra* Course

Each student in our *Modern Algebra* course was assigned a mathematician and mathematics to present to the rest of the class [9]. The course had many women students. Since women often relate more easily to women mathematicians, it was worthwhile to include women in the list of mathematicians. In addition, it was easy to do so, because there are numerous women who have done important work in this area. We chose topics that complemented course material and exposed the class to more current topics than are often presented in standard *Modern Algebra* courses, such as Sophie Germain and her work on Fermat's Last Theorem, Marjorie Lee Browne and her work on the matrix groups O(n) and U(n), and Emmy Noether and the ascending chain condition on sets and rings. While there is not normally enough time to cover these types of topics in this course, historical projects allow students to explore them without taking too much class time. Students presented biographies and mathematics after the related prerequisite course material was covered in class. For example, Sophie Germain's work on Fermat's Last Theorem was presented after modular arithmetic was covered in class.

Projects in a *Women and Minorities in Mathematics* Course

Presentations, worksheets and papers based on student projects were used to run a senior level seminar class on women and minorities in mathematics [8, 10]. The three assignments were divided up according to when the mathematician was born: (1) the 18th and 19th centuries, (2) between 1900 and 1925, and (3) after 1925. We gave the students references and directed questions for the first project, but as the students became more independent, with our guidance they chose their own mathematicians, found their own references and decided what mathematics they wished to focus on. Many future high school teachers took this class. This was an especially useful activity for them because as teachers they will routinely decide the level of mathematics that is appropriate for their classrooms. Most of the students were very motivated to study the related mathematics, perhaps because the methodology engaged them and they enjoyed the interdisciplinary nature of the subject area. The course was successful, and it ran a second time by student request.

Modeling the Worksheet Process in the *Introduction to Mathematics* Class

As students in the introductory mathematics course worked on their projects outside of class, in class we began the *What is a Mathematician?* segment by contrasting the diverse mathematical styles of Andrew Wiles and Carolyn Gordon.

We chose mathematicians whose research could be incorporated into a classroom without needing much background, and we specifically chose living mathematicians so that students would be forced to confront the commonly preconceived notion that mathematics is a dead field, with issues resolved long ago.

We chose Andrew Wiles, a white male mathematician, in order to take advantage of NOVA's *The Proof* video [25] on his solution of Fermat's Last Theorem. The video is extremely well done and addresses his influences, support, barriers, and mathematical style. In addition, there are many resources available to help incorporate his mathematics into classrooms [30]. Students viewed this video and then answered questions about Andrew Wiles related to the segment themes [16]. The next day, they compared their ideas with a worksheet that modeled a write-up of these themes and engaged them with related mathematics [11].

Since the video mentioned the fact that Sophie Germain was the only woman to work on Fermat's Last Theorem, and there were virtually no women seen in the video, we then examined detailed statistics on women and underrepresented minorities in mathematics [28, 32].

We wanted to contrast Andrew Wiles with a woman mathematician whose mathematics could be incorporated into the classroom. While we could have chosen Sophie Germain, we wanted to choose a living mathematician, so Sophie Germain was assigned to students instead. In addition, we wanted to choose as a model a woman who worked with many people, in order to contrast her with Andrew Wiles' solitary style, and we also wanted someone who was interested in gender issues, as we wanted to introduce the class to these issues. There are many possible choices of women mathematicians who satisfy these criteria, who also can be assigned as student projects.

We chose Carolyn Gordon, a well-known and respected geometer. While she is best known for her groundbreaking work on hearing the shape of a drum, she continues to do research as a leader in the related field of spectral geometry. Since there are only a few resources available for Carolyn Gordon and creativity is needed in order to engage students with ideas related to her mathematics, it is better to use her mathematics as a model for the class rather than as a student project topic. Carolyn Gordon satisfied our other criteria because she has many co-authors and is involved in AWM (Association for Women in Mathematics), and because she agreed to be interviewed for the worksheet so we could obtain information related to the segment themes. The contrasting styles of Carolyn Gordon and Andrew Wiles make them excellent choices for the beginning of the *What is a Mathematician?* segment.

Selections from the Andrew Wiles and Carolyn Gordon Worksheets

An informal style of writing is used in our worksheets on Andrew Wiles and Carolyn Gordon in order to encourage students to identify with the mathematicians and not be intimidated by the mathematics. As part of this informal style, we do not use a formal reference system, but we do give students a list of references at the end of each worksheet and comment on how we used them (see Appendix A).

Here we present the mathematical style section of the Andrew Wiles worksheet that we give to our students (see [11] and [25]).

Mathematical Style (from the Andrew Wiles Worksheet)

Andrew Wiles describes mathematical research as follows:

> Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. One goes into the first room, and it's dark, completely dark. One stumbles around bumping into the furniture, and gradually, you learn where each piece of furniture is, and finally, after six months or so, you find

> the light switch. You turn it on, and suddenly, it's all illuminated. You can see exactly where you were. Then you move into the next room and spend another six months in the dark. So each of these breakthroughs, while sometimes they're momentary, sometimes over a period of a day or two, they are the culmination of - and couldn't exist without - the many months of stumbling around in the dark that proceed them.

Andrew Wiles needs intense concentration in order to do mathematics. He reads books or articles to see how similar problems have been solved and he tries to modify other people's techniques to complete his own research. He uses by-hand calculations in order to look for patterns. When he is stuck on a problem, he tries to change it into a new version that is easier. Sometimes he must create brand new techniques to solve a problem and he is not sure where these come from. If he is stymied, he plays with his children or walks by the lake in order to relax and allow his subconscious to work.

While Andrew Wiles worked alone in secrecy for seven years, working with others has still been important to him. For example, before he publicized his proof of Fermat's Last Theorem, he first explained it to Nicholas Katz. After he could not fix the fundamental error later found in his proof, he called in Richard Taylor to help him. The two eventually fixed the problem. Hence, we see that while Wiles likes working alone, collaborative efforts have also been essential to his mathematics.

Andrew Wiles is a devoted and persistent mathematician. Even though he had no idea whether he could ever find a complete proof, especially because a proof of Fermat's Last Theorem might have required methods well beyond present day mathematics, he never gave up.

We wanted the students to engage the related mathematics. In the case of Andrew Wiles, it was easy to engage students with activities from NOVA's website [30]. For example, we used an activity where students demonstrate the Pythagorean Theorem by cutting out and matching up the relevant squares. Since the rest of the worksheet [11] was adapted from NOVA's website, we will not present it here.

Creating good classroom activities at the proper level can be a challenge if classroom resources on the mathematician and related mathematics are not available, as was the case with Carolyn Gordon's mathematics. Sometimes creativity is needed in order to engage the students. Here we present an in depth look at selections from the Carolyn Gordon worksheet and the activities we created to engage our students (see [12] and Appendix A).

Gender Issues (from the Carolyn Gordon Worksheet)

During the entire time that she was an undergraduate, and during her first couple of years as a graduate student, Carolyn Gordon never met another woman mathematician. In addition, her mathematics courses consisted almost exclusively of men. Yet she was not aware that this was an issue that bothered her until she attended a conference and went to an AWM gathering during her third year in graduate school. When she walked into that room full of women mathematicians, she was shocked by the experience and recognized her previously unacknowledged sense of isolation.

Today she balances a successful research and teaching career at Dartmouth College with her family. She is married to a mathematician, David Webb, and they have a daughter. In addition, she is heavily involved in AWM. Carolyn Gordon sees how AWM has helped women who have encountered barriers and active discrimination, both firsthand and through the experiences of friends and colleagues. She has seen the importance of role models, and has become one herself, mentoring many women students and young faculty.

Mathematical Style (from the Carolyn Gordon Worksheet)

Even though she is a geometer, she describes herself as being "terrible" at visualization and also as having a bad memory. While she is good with numbers, she says that this skill does not help in her research.

The following story also gives us insight into her mathematical style. The breakthrough that led to her research on hearing the shape of a drum occurred during her talk at a conference when a member of the audience asked her a related question. David Webb said that the question "was like a cold shower. It really made us sit up and think about this." Afterwards, the pair spent days making models and checking to see if they worked. Carolyn Gordon recalls, "We got these huge [paper] castles. They took up our living room."

According to Carolyn Gordon, sometimes she needs to step away from a problem and let her subconscious work. Then, while she is partially occupied with something else, new ideas will come to her. She compares the process of doing research to being in a maze. "Sometimes, when you are completely lost, you have taken a wrong turn, and you must back away and try a new direction. Other times, you will reach a door, find a way to open it, and discover that you have made progress and entered a deeper, more significant part of the maze."

Mathematics (from the Carolyn Gordon Worksheet)

During the 19th century, Auguste Comte, a French philosopher, hypothesized that the chemical composition of stars would always be beyond the reach of scientists. He was incorrect, and soon afterwards scientists developed the related field of spectroscopy. In spectroscopy, the pitches at which a molecule rings are used to attempt to identify the structure. Given a set of vibration frequencies, an important research topic is to ask what can be inferred about the object's composition.

In 1966, mathematician Mark Kac asked a related question about whether one can always hear the shape of a drum. In other words, if you close your eyes and listen to differently shaped drums being played, Mark Kac wanted to know if you could distinguish the shape by the sound or vibration frequencies you hear. A mathematical drum is not a standard musical instrument; it is any shape in the plane that has an interior and a boundary, such as a circle, a square, or a triangle. The interior vibrates with each strike while the boundary frame remains rigid. Imagine that we had a machine that could tell us the exact frequency of the sound of the drum vibrations. Then we could check and see whether the machine could always distinguish the sounds of differently shaped drums.

In 1911, Hermann Weyl proved that one can always hear the area of a mathematical drum. It makes sense that we can hear the area since the bigger the area of the drum, the lower the tone. Later, in 1949, Subbaramiah Minakshisundaram and Ake Pleijel proved that one can always hear the length of the boundary, or the perimeter of the drum. It was then thought that the sound of a drum might determine its shape, and Mark Kac asked whether this was true.

The problem challenged researchers. Finally, in the spring of 1991, Carolyn Gordon, her husband David Webb, and Scott Wolpert, the audience member that we previously mentioned, came up with the answer: No! One can sometimes, but not always hear the shape of a drum. They found two mathematical drums that have different shapes, but still had the same vibration frequencies, therefore making exactly the same sound.

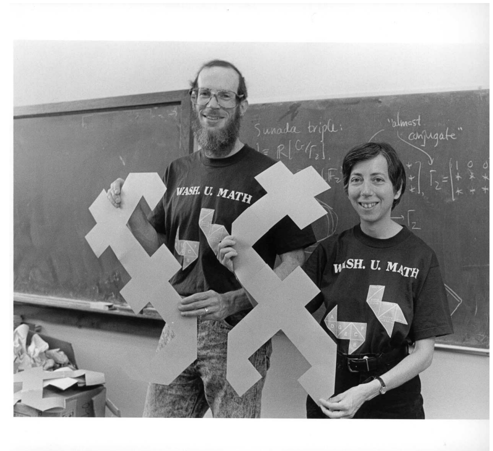
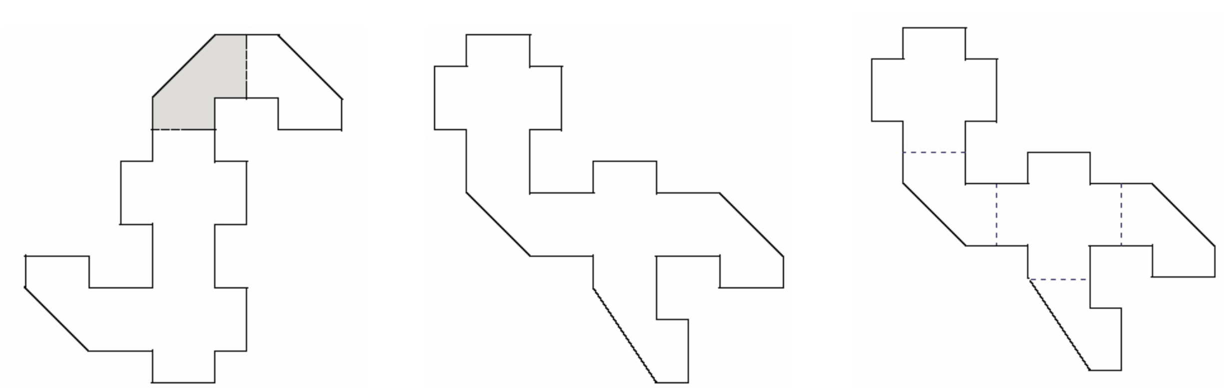


Figure 1: Carolyn Gordon and husband David Webb with their drums in 1991. Printed with permission of Washington University.

Mathematicians would not have been satisfied with experiments to show that the drums sound the same because calculations and experiments cannot be exact and a very small frequency difference could escape experimental detection. Instead, Carolyn Gordon and her collaborators used mathematics to prove that her drums sounded the same without actually testing them in real-life. Later on, physicists created the drums and tested them in real life. They found that the drums sounded nearly the same with an error attributed to the experimental procedures.

Classroom Activities (from the Carolyn Gordon Worksheet)



Worksheet Diagram 2 **Worksheet Diagram 3a and 3b** **Worksheet Diagram 3c**

Figure 2: Drums for Classroom Activities. Reprinted with Permission of Ivars Peterson.

1) Cut along the boundaries of the sound-alike drums in Diagrams 3b and 3c (see Figure 2) which are duplicate copies of Diagram 3a. If the drums in Diagrams 2 and 3a have

the same shape, then you will be able to place one on top of the other. Place Diagram 3b on top of Diagram 2 above, and try to move Diagram 3b (via rotating, translating or flipping it around) so that it matches Diagram 2. Question: Do these figures have the same shape?

2) Take Diagram 3c (the one with the dashed lines marked on it) and cut this drum along the dashes. Notice that you will have 5 pieces total. Try to fit these cut pieces onto Diagram 2. Notice that you won't be able to do so. In fact, no matter how you might cut Diagram 3c (don't try this now), you won't be able to fit it onto Diagram 2. This contradicts the fact that they must have the same area in order to sound the same. Let's try and resolve the apparent conflict by investigating the accuracy of the models represented. Identify which piece does not fit properly onto Diagram 2 above. Put a star on this piece on Diagram 3a and also put a star on this piece on Diagram 3b, which is the drum that you cut out along the boundary but left in one piece.

3) Take Diagram 3b and compare it to the drum Carolyn is holding (see Figure 1). Compare the piece that you starred with the same piece on Carolyn's drum. Notice that the vertical edge next to the part that Carolyn is holding is the same length as the vertical edge opposite it, but that this is not true of your starred piece and its opposite edge. The drum that Carolyn is holding is drawn correctly, but the starred piece on Diagrams 3a and 3b was not correctly drawn to scale and it is this error in scaling that causes the contradiction in 2). In Diagram 3a above, try to fix the problem piece and edge so that it is drawn to scale by adding to Diagram 3a to show how you would have drawn the piece. Take one of the other similar but correctly scaled pieces, place it on top of the problem piece and trace the correctly scaled piece in order to fix the problem piece.

The point of this exercise is to have you engage the models instead of just hearing about them (no pun intended). There are dangers in relying on models since it is difficult to create physical models representing abstract figures with precisely determined sides. These models were found on a webpage that discussed Carolyn Gordon's solution of the problem and the physicists' work that followed. The drums in the picture of Carolyn Gordon and David Webb are drawn correctly, and they have the same area and perimeter, but are shaped differently. If you close your eyes and listen to them being played, you cannot tell that they have different shapes, since they sound exactly the same.

Carolyn Gordon's research answers Mark Kac's question, but it raises many new issues. For example, now that we know that one cannot hear every property of a drum, what properties besides its area and perimeter really are audible? In addition, we know a great deal more about the vibrations of sound than about the vibrations of light. In the field of spectroscopy, we hope to recover the chemical composition of stars from their vibration fingerprints. Carolyn Gordon's research on hearing the shape of a drum is a small step in this direction. It also shows us that a mathematical proof does not need to be constructive and that there is not always just one conclusion that can be reached from a complete set of measurements.

Student Reactions to the Andrew Wiles and Carolyn Gordon Worksheets

Students report that they enjoy the worksheets and the activities and that they find these extremely helpful as models for their own worksheet. Students respond well to the openness of Andrew Wiles and Carolyn Gordon about their mathematical styles and struggles, including Carolyn Gordon's feelings about gender issues in mathematics. The statistics on women and minorities then feel more personal to them. In order to cultivate this identification with gender issues, students read an article on the web called *Fifty-Five Cultural Reasons Why Too Few Women Win at Mathematics* [22].

At the beginning of the semester, students were asked to define mathematics. Since many of them reported that mathematics is the study of numbers or equations, these activities force them to re-examine their notion of what mathematics is. Student presentations begin during the week following the introduction of these worksheets. Once students have confronted their preconceived notions about mathematicians and mathematics, the rest of the mathematician segment is ordered so that we move forward in time to trace the historical progression of mathematics and the changing roles of women and minorities.

Applying the Worksheet Process across a Variety of Courses: Using the Example of David Blackwell

We will discuss some ideas for incorporating David Blackwell and his mathematics into two different levels of college courses.

David Blackwell is considered to be one of the greatest African-American mathematicians [31], and yet, perhaps because he is still alive, resources on minorities in mathematics that are designed for use in the classroom do not include him. This is unfortunate since students relate to Blackwell's openness about racial issues and his mathematical style. In addition, Blackwell has done work in several fields such as game theory, Markov matrices, statistics and probability, and so ideas related to his mathematics can easily be incorporated into a variety of levels of classes [18].

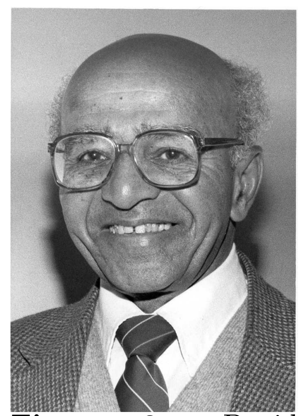


Figure 3: David Blackwell. Printed with permission.

David Blackwell Project in an *Introduction to Mathematics* Course

David Blackwell is the first living mathematician presented in this class as a student project. Students enjoy learning about him and his mathematics enough that they report this in their evaluations.

After class discussions on strategies for zero-sum games, where there is a clear winner and loser, we wanted to expose the students to the complexity of other types of games. In one interview [1, pages 25–26], David Blackwell discusses the Prisoner's Dilemma and its relationship to the arms race with the Soviet Union, so we created a list of related mathematics questions for the students to answer in their project.

Directed Mathematics for David Blackwell

What is game theory?
What is the Prisoner's Dilemma? How does this relate to what David Blackwell worked on?
What matrix of payoffs represents the Prisoner's Dilemma?
If a person is deciding what to do, why does it make sense (when looking at the possible cases) for him to confess?
What are the applications of game theory to real life?

We provided the students with the resources that they would need to complete this project (see Appendix B). Students do struggle with the idea of a selfish strategy in the Prisoner's Dilemma, as opposed to a cooperative strategy. In one class, a student struggling with these ideas stated, "If I were a criminal and was going to rob a bank with someone, then I would make sure that he was honest and that I could trust him not to turn me in." When we pointed out the student had just used the words "criminal" and "honest" in the same sentence, the class laughed and then re-examined their beliefs. The arms race with the Soviet Union, which David Blackwell discussed in the interview mentioned above, is especially effective at helping students differentiate between the various strategies.

Sample Student Worksheet for David Blackwell

The following classroom worksheet was completed by students Ross Bryant and Zachary Lesch-Huie from the Fall 2001 *Introduction to Mathematics* course. This worksheet is a nice response to the assignment. Since they used our Andrew Wiles and Carolyn Gordon worksheets as models, they turned in a list of references with comments (see Appendix C) instead of using a formal reference system. We have left the worksheet in the form turned in by the students in order to provide a true sample of student work as they handed it out to the rest of the class, including the minor errors contained within.

Introduction (from the David Blackwell Student Worksheet)

David Blackwell is the first truly contemporary mathematician the class will study. Historical accounts of person's lives are doomed to appear hazy with age. Blackwell is interesting for this reason, because he is the only person thus far whose actions are not obscured by time. We have been able to look over Blackwell's own words through interviews to get a personal perspective on his incredible life and work. For one so accomplished, Blackwell certainly comes across as a man who is truly humble, genteel, staggeringly intelligent, and deeply in love with his work. At an early age Blackwell discovered an ability and love of math, and with the support of family and teachers he eventually rose to become the honored expert in statistics and professor at the University of Berkeley that he is today. His main interest has always been investigating and teaching set theory and probability theory. He has also made significant contributions to Bayesian statistics and game theory. Blackwell has been awarded the von Neuman Prize by the Operations Research Society of America, and in 1986, he received the most prestigious award in the field of statistics, the R.A. Fisher Award. He also has the distinction of being the first African-American mathematician elected to the National Academy of Sciences in 1965. In this report, we will explore these aspects of Blackwell's life: influences and support, issues of race, mathematical style, and game theory.

Influences and Support (from the David Blackwell Student Worksheet)

Blackwell traces his mathematical abilities to his grandfather. While Blackwell never knew him, he was inspired by his legacy: it was in the library of books that his grandfather left to his family that Blackwell found his first algebra book. He also remembers an uncle who could add three columns of numbers in one step. His parents were quite supportive of his math and had hopes that he would become a grade school teacher. His ability however, would see him surpass these aspirations in ways his parents never dreamed of. In high school his mathematics club advisor would give the students problems from the School Science and Mathematics journal and submit their solutions. Blackwell was published. Blackwell entered college at the tender age of 16 and achieved a BA in a mere 3 years. It was during his graduate studies at Illinois University that Blackwell met his most important mathematical influence Joe Doob. It was Doob who directed Blackwell's research and steered him in the direction probability theory. Later, in 1945 Blackwell claims he got started in statistics after listening to one lecture by Abe Girshick. At the time of the lecture Dr. Abraham Girshick was an already famous mathematician, but Blackwell thought that he had caught a mistake in his lecture. Blackwell wrote to him describing the error. Though the supposed error turned out to be unfounded, the newcomer impressed Girshick, and the two began a partnership. They wrote a book on statistical theory which was published in 1954.

Racial Issues (from the David Blackwell Student Worksheet)

When one reads of Blackwell's childhood it becomes clear how hard his family tried to shelter him from the prejudice of early twentieth-century America. The family had had their share of difficulty; the reason his fast-adding uncle never went to school was because his grandfather feared he would be mistreated because of his race. Though the grade school Blackwell attended in Centralia, Illinois was integrated, there were two segregated schools in the same town. Blackwell makes clear he didn't feel discriminated against early on, but this was a result of his parent's protection. Still, racist culture continued as a presence in Blackwell's life. Even when he had proved himself to brilliant mathematical student there was tension. When he was being considered for membership in the Institute of Advanced Study it was customary for members of the Institute to be appointed honorary members of the faculty at Princeton. Princeton objected to a black man being an honorary member of their faculty. The Director of the Institute objected on Blackwell's behalf, and Princeton backed down. Even this struggle was only partly known to Blackwell at the time. Says Blackwell: "Apparently there was quite a fuss over this, but I didn't hear a word about it." Even after Blackwell received his Ph.D. from Illinois (at age 21) his aspirations were limited by the culture of racism. "It never occurred to me to think about teaching in a major university since it wasn't in my [racial] horizon at all." Blackwell fired off 105 applications, one to every black college at the time. He accepted the first offer he got, from Southern University in Louisiana, and later moved to Clark College in Atlanta, and finally to Howard University in Washington, D.C., the top Black institution at the time. In 1950 Blackwell took a leave of absence to work at the RAND Corporation. In 1954, he was asked to join the faculty of the University of California at Berkeley. The position was similar to one he had been interviewed for some years before, when his race had been an issue.

Mathematical Style (from the David Blackwell Student Worksheet)

Blackwell describes himself as a mathematical dabbler. "I've worked in so many areas - I am sort of a dilettante." To some this may imply that he lacks the ability to focus, but this is simply not the case. He is in fact well known for presenting his theorems in elegantly clear ways. Perhaps this is because of his slightly different way of appreciating math. For example, instead of describing the process of math technically, he uses more common, almost sentimental language to describe his own mathematical aesthetic. "There is beauty in mathematics at all levels" he explains. "The whole subject [is] just beautiful." Blackwell is also one of the voices of a radical type of statistics known as Bayesian statistics. Whereas "Classicist" statisticians see probabilities as entirely objective, Bayesians believe that there is always a suspected set of probabilities in the mind of the experimenter. When tackling a math problem, Blackwell finds that his main motivation is for understanding, a desire particularly meaningful to him. "I am not interested in doing research and I never have been. I'm interested in understanding, which is quite a different thing," he says. This rather broad desire often places Blackwell in a wide range of mathematics (which accounts for his wide range of contributions), filling in "holes" in certain theories so that his understanding can be "rounded out." By no means does this limit him to working alone. In fact, MathSciNet showed 90 published papers with Blackwell's name on them many of them were with one or more partners. Indeed, in his search for understanding, Blackwell finds both working alone and with others helpful.

The Math (from the David Blackwell Student Worksheet)

It was while working at RAND and Girshick that Blackwell first began working with statistics and probability theory. These areas of interest led him into the field of game theory, the subject that he is perhaps best known for. Game theory is a branch of statistics in which game-like situations are "played" out with specific objectives. "The decision making options of the players are then statistically analyzed." Blackwell applied game theory most famously to old-style pistol duels. If two armed people are advancing on one another, when should they shoot? While Blackwell investigated the problem, a RAND economist consulted him about drawing up a budget proposal for the coming Cold War years. Was war imminent or not? The economist wanted to know. If so, they should begin budgeting for a short-term solution to the Soviet "problem." Blackwell's hypothetical "duelists" had just been placed in the context of World War 3. Blackwell, of course could not provide an answer, but found the problem none-the-less troubling. RAND saw the real-world application of game theory as a means to outwit their political opponent. However, game theory is for the most part much more benign, and can be used to understand and explore methods of decision-making and problem solving. Consider the classic game theory scenario called "The Prisoner's Dilemma:"

The Prisoner's Dilemma (from the David Blackwell Student Worksheet)

Imagine that two criminals, Zach and Ross, have raided a farmer's grain silo in a fiendish attempt to make off with the farmer's sweet, sweet sorghum. The two are apprehended by the police as they leave the scene of the crime, separated at the station house, and shaken down by the coppers. Each crook has to choose whether or not to confess and implicate the other. If neither man confesses, they both will serve one year on a charge of possessing illicit sorghum. If each confesses and implicates the other, both will go to prison for 10 years. However, if one crook confesses and implicates the other and the other crook does not confess, the one who has ratted out his accomplice with go free, while the other crook will rot in the pokey for 20 years on the maximum charge.

The strategies of the "players" or crooks are: confess or don't confess.

The payoffs (in this case, penalties) are the sentences served.

In game theory, data is expressed in a "payoff table" (or payoff matrix). Here is the payoff table for the Prisoner's Dilemma:

		Ross	
		Confess	Don't Confess
Zach	Confess	$10, 10$	$20, 0$
	Don't Confess	$0, 20$	$1, 1$

Both crooks choose one of the two strategies. Or, Zach chooses a column and Ross chooses a row. The two numbers in each cell represent the years in jail for the prisoners when those two strategies are chosen. In each cell (a,b), a represents the amount of jail time for Zach, and "b" represents the amount of jail time for Ross.

Solution: One must go inside the mind of the prisoner. Zach might think: "Two things can happen, Ross can rat or clam up. If Ross confesses, I get 20 years if I don't confess, 10 years if I do, so in that case it's better to confess. Then again, if Ross stays quiet, and I do too I get a year; but in that case, if I do confess I can go free. Either way, it's best if I confess. So I'll confess." One can assume that Ross' line of thinking will go the same way. So if they both act "rationally" and confess they both go to prison for 10 years, putting up with horrible abuse and all for a sack of sorghum. Here's the thing, if they had both acted "IRRATIONALLY," and kept quiet, they could have gotten off with one year each, and in minimum security too. When both prisoners confess, they have fallen into what is called Dominant Strategy Equilibrium. Because each crook chooses the option with the best payoff, they choose the "dominant" strategy. But in this case these dominant strategies have made both the prisoners worse off. This is an interaction that can be applied to many aspects of modern life. Arms races, pollution, over hunting, ect, are all instances where (it seems) the individually rational action leads to horrible results for each person.

Class Exercises (from the David Blackwell Student Worksheet)

1. Break the class in half. Then break each half into pairs. Each member of a pair is a "player" in The Prisoners' Dilemma. One half of the class' prisoners are "separated" and cannot communicate with one another. The other half of the class has a chance to talk together before they make a decision. Write down whether you can talk with your partner or not.
2. Given the circumstances of the payoff table, what will you choose to do, confess or hold out? Why?
3. Relate this information with what you learned about the nature of the Prisoners' Dilemma and Dominant Strategy Equilibriums at the end of the presentation.

We encourage the rest of the class to be skeptical learners as they are reading the worksheet and completing the activities, and we ask them for suggestions for improvement. This is especially important because even though we ask students to turn in preliminary responses to the mathematics questions and thematic issues in order to receive feedback before their presentations, errors may still appear in the presentations and worksheets. While this student worksheet contains only some minor errors, in some other cases students present or hand out work containing major errors. Hence, it is essential for the instructor to go over the mathematics and correct any historical or mathematical inaccuracies after each presentation.

David Blackwell Project in a *Women and Minorities in Mathematics* Course

David Blackwell and his mathematics can also be incorporated into higher level classes. In the *Women and Minorities in Mathematics* course, mathematics questions and resources were given to students for the first paper assignment in the course that included mathematicians born before 1900. Since David Blackwell was born in 1919, he was a possible choice for the second paper assignment, which focused on mathematicians born between 1900 and 1925. We chose this time frame because both women and minority mathematicians were just beginning to gain acceptance but shared common issues in the context of society at the time. Since we wanted students to develop their own research skills, with our help, they chose their own mathematician, found their own references and decided what mathematics they wished to focus on.

One student chose David Blackwell. Since he has published over 90 papers and books in various areas of mathematics, there were many possible related mathematical topics to choose from. She chose to discuss the idea of a Markov matrix, which was the topic of Blackwell's doctoral thesis [2], and she also chose to explore a paper of Blackwell's called *The Big Match* [3], in which Blackwell looks at an infinite game, that is, a game with an infinite numbers of moves. Since Blackwell also discusses infinite games in [6, pages 46–47], this was a good choice for a topic. The student created a presentation, classroom activity sheet and a paper [20]. In the process she learned a lot of mathematics along with the context of the thematic issues.

Student Responses to David Blackwell Projects

Students admire Blackwell's attitudes about racial issues and his mathematical style [1, 6] and they enjoy learning about him and his mathematics. In the *Introduction to Mathematics* class, where each student wrote about a mathematician that he or she related to, one female student commented:

> I never thought that I could relate my mathematical style to a famous mathematician because first of all I didn't really think that I had a mathematical style. Doesn't having a mathematical style assume that one is greatly interested in mathematics? Well, not in my case. Math really has not been one of my strong points. Although I am interested in solving problems, I'm more concerned with understanding what I'm learning. I finally found a mathematician that I can identify with and that person is David Blackwell.

Conclusion

Projects that incorporate the mathematical achievements of women and minorities into the classroom provide a rich mathematical environment for examining beliefs about what mathematics is and the diverse ways that people can be successful at doing mathematics. Because most women and minority mathematicians that we know about lived within the last 200 years, there are not as many resources that bring their mathematics into the college classroom in a meaningful way as there might be for earlier mathematicians. Yet the inclusion of the recent mathematical achievements of women and minorities is important, not only to ensure that these mathematicians are remembered for their mathematics instead of just their race or gender, but also because students relate to these mathematicians. With creative planning, their achievements can be incorporated into the classroom, and the result can be rewarding for all students.

One student from the *Women and Minorities in Mathematics* class, who planned to become a high school mathematics teacher, commented that this was one of the few classes in the department that he could walk away from and actually say, "I will use this in my classroom." Another student commented that this course should be taken by both math majors and future teachers, while a third student commented that she felt the projects helped her in advancing her ability to do research and speak about math, and that "it was very encouraging." Students from this class are now teaching in the middle grades and high school and have reported successful implementation of projects on women and minority mathematicians in their own classrooms.

One *Introduction to Mathematics* student commented, "Some students might say that learning about present and past mathematicians is useless, however, I learned that not all mathematicians sit at a desk doing equations. Some, such as Gordon, work on drums, while others, such as Blackwell, work with theories such as game theory." Another student reflected, "I thought that it was great that my mathematical style was so close to hers. It made me appreciate her way of thinking better, as well as my own way of thinking. I had never before really labeled my train of thought, but because of our studies I am aware of the way my mind works, and I find it very handy when I come across difficulties that I must deal with. Thanks!" The strength of this reaction is exactly the reason that we use these projects. This is the kind of response that we hope for in all of our students.

Acknowledgements

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References

1. D.J. Albers and G. Alexanderson, "David Blackwell," in *Mathematical People: Profiles and Interviews*, D.J. Albers and G. Alexanderson, eds., Birkhäuser, Boston, MA, 1985.

2. David Blackwell, *Some Properties of Markov Chains*, Ph.D. thesis, University of Illinois-Urbana Champaign, 1941.

3. D. Blackwell and T.S. Ferguson, "The Big Match," *Annals of Mathematical Statistics*, 39 (1968) 159–163.

4. M.A. Campbell and R. Campbell-Wright, "Towards a feminist algebra," in *Teaching the Majority: Breaking the Gender Barrier in Science, Mathematics and Engineering*, Sue Rosser, ed., Teachers College Press, Columbia University, New York, 1995.

5. C. Davis and S. Rosser, “Program and curricular interventions,” in *The Equity Equation: Fostering the Advancement of Women in the Sciences, Mathematics, and Engineering*, Cinda-Sue Davis et al, ed., Jossey-Bass Publishers, San Francisco, CA, 1996.

6. Morris DeGroot, “A Conversation with David Blackwell,” *Statistical Science*, 1 (1986) 40–53.

7. William Dunham, *The Mathematical Universe*, John Wiley & Sons, New York, 1994.

8. Sarah J. Greenwald, *Women in Math Course* [online], 1999, Available: http://www.mathsci.appstate.edu/∼sjg/womeninmath/.

9. ———, *Modern Algebra* Course [online], 2000, Available: http://www.mathsci.appstate.edu/∼sjg/class/3110/.

10. ———, *Women and Minorities in Math Course* [online], 2001, Available: http://www.mathsci.appstate.edu/∼sjg/wmm/.

11. ———, *Andrew Wiles and Fermat’s Last Theorem Worksheet* [online], 2002, Available: http://www.mathsci.appstate.edu/∼sjg/class/1010/mathematician/wiles2.doc.

12. ———, *Carolyn Gordon and Hearing the Shape of a Drum Worksheet* [online], 2002, Available: http://www.mathsci.appstate.edu/∼sjg/class/1010/mathematician/carolyngordon.pdf.

13. ———, *Mathematician Segment* [online], 2002, Available: http://www.mathsci.appstate.edu/∼sjg/class/1010/mathematician/.

14. ———, *Mathematician Segment Presentation Checklist* [online], 2002, Available: http://www.mathsci.appstate.edu/∼sjg/class/1010/presentationchecklist.html.

15. ———, *Mathematician Segment References* [online], 2002, Available: http://www.mathsci.appstate.edu/∼sjg/class/1010/mathematicianreferences.html.

16. ———, *Mathematician Segment Theme Questions on Andrew Wiles* [online], 2002, Available: http://www.mathsci.appstate.edu/∼sjg/class/1010/mathematician/wiles2.doc.

17. ———, *Mathematician Segment Worksheet Checklist* [online], 2002, Available: http://www.mathsci.appstate.edu/∼sjg/class/1010/worksheetchecklist.html.

18. ———, *NCCTM Centroid Column and Activity Sheets on David Blackwell and Game Theory* [online], 2003, Available: http://www.mathsci.appstate.edu/∼sjg/ncctm/activities/.

19. Art Johnson, *Famous Problems and their Mathematicians*, Teacher Ideas Press, Englewood, CO, 1999.

20. Elizabeth Johnson, *David Blackwell* [online], 2001, Available: http://www.mathsci.appstate.edu/∼sjg/wmm/final/blackwellfinal/main.htm.

21. Karen Karp, *Feisty Females: Inspiring Girls to Think Mathematically*, Heinemann, Portsmouth, NH, 1998.

22. Patricia C. Kenschaft, “Fifty-five cultural reasons why too few women win at mathematics,” in *Winning Women into Mathematics*, P.C. Kenschaft and S. Keith, eds., Mathematical Association of America, Washington, DC, 1991.

23. Beatrice Lumpkin, *Geometry Activities from Many Cultures*, J. Weston Walch, Portland, ME, 1997.

24. B. Lumpkin and D. Strong, *Multicultural Science and Math Connections: Middle School Projects and Activities*, J. Weston Walch, Portland, ME, 1995.

25. NOVA, *The Proof* [video], 1997.

26. Marla Parker (Ed.), *She Does Math! Real-Life Problems from Women in the Job*, Mathematical Association of America, Washington, DC, 1995.

27. Teri Perl, *Math Equals: Biographies of Women Mathematicians + Related Activities*, Addison-Wesley, Menlo Park, CA, 1978.

28. J.E. Taylor and S.M. Wiegand, *AWM in the 1990s: A Recent History of the Association for Women in Mathematics* [online], 2003, Available: http://www.awm-math.org/articles/199812/awm1990s/.

29. Janet Trentacosta (Ed.), *Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity*, National Council of Teachers of Mathematics, Reston, VA, 1997.

30. WGBH Science Unit, *NOVA Online - The Proof* [online], 1997, Available: http://www.pbs.org/wgbh/nova/proof/.

31. Scott Williams, *Mathematicians of the African Diaspora: Who are the Greatest Black Mathematicians?* [online], 2002, Available: http://www.math.buffalo.edu/mad/madgreatest.html.

32. ———, *Mathematicians of the African Diaspora: Statistics on the Numbers of Blacks Receiving Mathematics Ph.D.s* [online], 2003, Available: http://www.math.buffalo.edu/mad/stats/index.html.

Appendix A: References and Comments on How We Used Them for Carolyn Gordon

We do not use a formal reference system for worksheets, but we do give students a list of references at the end of each worksheet and comment on how we used them in designing the worksheet. We gave the following references to the *Introduction to Mathematics* class at the end of the Carolyn Gordon worksheet.

Cipra, Barry. (1992). You can't always hear the shape of a drum. Science, March 27, 1992, Volume 255, No. 5052, p. 1642 –1643. This magazine article had an overview of the problem and a description of the reaction to Wolperts question, their model building, and transatlantic work.

Cipra, Barry. (1997a). You can't always hear the shape of a drum [On-line]. Available: http://www.ams.org/new-in-math/hap-drum/hap-drum.html
This is a great website that I used to find an overview of the history and solution of the problem.

Gordon, Carolyn and David Webb. You Can't Hear the Shape of a Drum. American Scientist, January-February, 1996, Volume 84, No. 1, p. 46 – 55.

This magazine article had a great summary of the solution of the problem, and some information about the authors. They recently received The Chauvenet Prize for writing this article, given for an outstanding expository article on a mathematical topic by a member of the Mathematical Association of America.

Mathscinet search on Carolyn Gordon. (2001) [On-line].
Available: http://www.ams.org/mathscinet
I used this site to find her published papers and collaborators.

Personal communication with Carolyn Gordon (2001).

Peterson, Ivars. (1997a). Ivars Peterson's MathLand: Drums that sound alike [On-line]. Available: http://www.maa.org/mathland/mathland_4_14.html I used this site to find the pictures in Figure 3 and 4, and it also contained information about the physicists who made the drums and performed experiments to show that they sounded the same.

Weintraub, Steven. (1997). What's new in mathematics June 1997 cover [On-line]. Available: http://www.ams.org/new-in-math/cover/199706.html
This website contains the pictures of Carolyn Gordon and David Webb holding their drums. It also contains links to an animated picture of the frequency and waves when the drums are struck.

I could not find information in books about Carolyn Gordon.

Appendix B: References Provided to the Students for David Blackwell

We gave the following resources to the students assigned to David Blackwell in the *Introduction to Mathematics* class.

Albers, D. and Alexanderson, G. eds., Mathematical People: Profiles and Interviews, Birkhäuser, Boston,1985, p. 19-32.

DeGroot, Morris. "A Conversation with David Blackwell," *Statistical Science*, 1 (1986) 40-53.

O'Donnell, Michael. The Prisoner's Dilemma: A Fable [online] (1998).
Available: http://www.classes.cs.uchicago.edu/classes/archive/1998/fall/CS105/Project/node2.html

Williams, Scott. David Blackwell - Mathematicians of the African Diaspora [online] (2002).
Available: http://www.math.buffalo.edu/mad/PEEPS/blackwell_david.html

Young, Robyn. "David Blackwell" in *Notable Mathematicians : From Ancient Times to the Present* p. 62-64.

Appendix C: Reference Section from the Student Worksheet

Students Ross Bryant and Zachary Lesch-Huie submitted the following references at the end of their David Blackwell worksheet.

Albers, D. and Alexanderson, G. eds., Mathematical People: Profiles and Interviews, Birkhäuser, Boston,1985, p. 19-32.
This gave us background insight on Blackwell's life as a young man as well as Blackwell's feelings about working on the Prisoner's Dilemma for the RAND corporation and the relationship to the arms race with the Soviet Union.

DeGroot, Morris. "A Conversation with David Blackwell," *Statistical Science*, 1 (1986) 40-53.
This interview revealed Blackwell's personal experiences and dreams as a young man. He also discusses his mathematical techniques and inspirations.

O'Donnell, Michael. The Prisoner's Dilemma: A Fable [online] (1998). Available:
http://www.classes.cs.uchicago.edu/classes/archive/1998/fall/CS105/Project/node2.html
This website focused on Tucker and the Prisoner's Dilemma and is where we got most of our information on this subject and its significance.

Williams, Scott. David Blackwell - Mathematicians of the African Diaspora [online] (2002).
Available: http://www.math.buffalo.edu/mad/PEEPS/blackwell_david.html
We learned all about Blackwell's research and game theory at this website.

Young, Robyn. "David Blackwell" in *Notable Mathematicians : From Ancient Times to the Present* p. 62-64.
This selection helped in giving us a general overview of Blackwell's life and some insight regarding his work with the theory of duels and game theory in general.