Mat 4310          Exam 2 Review Topics

This is a basic review and some stuff I think is interesting.

Chapter 4: Polynomial interpolation: Know what it is and how to find the polynomial in both Newton and Lagrange form. Understand the theorem on existence and uniqueness, and Theorem 2 on page 174 giving a bound on the error in polynomial interpolation and how to use it.

Chapter #5: Know the basic trapezoid rule approximation and Simpson’s rule approximation (for 1 subinterval) and the composite rules for \( n \) equally spaced points. Know the error terms for each and how to use them to find a bound on the error as well as the number of intervals.

Know how to find an interpolating polynomial and to determine a quadrature formula from it. Understand how to use undetermined coefficients to find the best nodes and weights for the formula. (Make sure you know what best means in the previous sentence).

Exam 2 Review Questions

1.) Consider the simple quadrature rule on \([0,1]\) derived as

\[
\int_0^1 f(x) \, dx \approx \int_0^1 p(x) \, dx
\]

where \( p(x) \) is the linear polynomial interpolating \( f(x) \) at \( x = 0 \) and \( x = 3/4 \).

   a.) Find the linear interpolating polynomial and use it to write down this quadrature rule in the usual form (a weighted sum of values of \( f(x) \)).

   b.) Is this rule exact for linear polynomials? Quadratics? Cubics?

2.) Use the method of undetermined coefficients to find weights \( w_i \) and node \( a \) such that the rule

\[
\int_0^\pi f(t) \cos t \, dt = w_1 f(0) + w_2 f(a).
\]

is exact for polynomials of as high a degree as possible. What is this highest degree? (make sure there is none higher)

(Useful formulas: \( \int_0^\pi \cos t \, dt = 0 \) \( \int_0^\pi t \cos t \, dt = -2 \) \( \int_0^\pi t^2 \cos t \, dt = -2\pi \)

\[
\int_0^\pi t^3 \cos t \, dt = -3\pi^2 + 12.
\]

Use your formula to find an approximation to \( \int_0^\pi e^{-t^2} \cos t \, dt \).