Problem Session – Green  January 23, 2006   NAME: _____________________

Let $\vec{P}$ be the vector-valued function $\vec{P}(t) = (2\cos t, \sin t)$.

a. If $x = 2\cos t$ and $y = \sin t$, find an equation relating $x$ and $y$ (Hint: square $x$ and add the square of 2 times $y$ together).

b. Sketch the curve traced by $\vec{P}$ and draw the vector $\vec{P}'\left(\frac{\pi}{6}\right)$ on the point corresponding to $t = \frac{\pi}{6}$ on your picture (try to approximate the correct length).

c. Find a vector equation for the line tangent to the curve at $t = \frac{\pi}{6}$.

d. **Using the parametrization**, set up an integral to compute the length of the curve if $0 \leq t \leq 2\pi$. (Do not try to evaluate this integral).

From Calculus 2, we know that if $p(t), v(t), a(t)$ are a particle’s position, velocity, and acceleration at time $t$, then the following relationships hold:

$$v(t) = p'(t) \quad \text{and} \quad a(t) = v'(t) \quad \text{as well as} \quad p(t) = \int v(t) \, dt + p(0) \quad \text{and} \quad v(t) = \int a(t) \, dt + v(0)$$

The same hold true for vector-valued functions: If $\vec{p}(t), \vec{v}(t), \vec{a}(t)$ are a particle’s vector-valued position, velocity, and acceleration at time $t$, then the following relationships hold:

$$\vec{v}(t) = \vec{p}'(t) \quad \text{and} \quad \vec{a}(t) = \vec{v}'(t) \quad \text{as well as} \quad \vec{p}(t) = \int \vec{v}(t) \, dt + \vec{p}(0) \quad \text{and} \quad \vec{v}(t) = \int \vec{a}(t) \, dt + \vec{v}(0)$$
At time $t=0$ seconds, a particle is at the origin and has velocity vector $(4,4)$. It undergoes constant acceleration $\vec{a}(t) = (0, -1)$. (So what are $\vec{p}(0)$ and $\vec{v}(0)$?)

a. Using the equations at the bottom of the front page, find formulas for the velocity function $\vec{v}(t)$ and the position function $\vec{p}(t)$.

b. Write the Maple command to plot the path taken by the particle $\vec{p}(t)$ for $0 \leq t \leq 10$.

c. Eliminate the variable $t$ in the parametric equations for the curve $\vec{p}(t)$ to find a relationship between $x$ and $y$.

d. Set up an integral that gives the arclength of the curve $\vec{p}(t)$ for $0 \leq t \leq 10$.

Extra: Give the vector-valued function $\vec{p}(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq 4\pi$.

Find the vector equation for the tangent line $\ell$ to $\vec{p}(t)$ at $t = \pi$. 