**Cross Product**

**Defn** Let \( \vec{v} = (v_1, v_2, v_3) \) and \( \vec{w} = (w_1, w_2, w_3) \) be vectors in \( \mathbb{R}^3 \). Their **cross product** is the vector

\[ \vec{v} \times \vec{w} = (v_2 w_3 - v_3 w_2, v_3 w_1 - v_1 w_3, v_1 w_2 - v_2 w_1) \]

**Properties:**
1. The cross product is **only** defined for vectors in \( \mathbb{R}^3 \).
2. The cross product is **not** commutative \( \vec{v} \times \vec{w} = -\vec{w} \times \vec{v} \).
3. The vector \( \vec{v} \times \vec{w} \) is perpendicular to both \( \vec{v} \) and \( \vec{w} \).
4. \( \vec{v} \times \vec{v} = \vec{0} \).
5. \( \vec{i} \times \vec{j} = \vec{k}, \ \vec{j} \times \vec{k} = \vec{i}, \ \text{and} \ \vec{k} \times \vec{i} = \vec{j} \).

Remembering the cross-product.

**Determinant of the matrix** OR

\[
\begin{bmatrix}
\vec{i} & \vec{j} & \vec{k} \\
v_1 & v_2 & v_3 \\
w_1 & w_2 & w_3
\end{bmatrix}
\]

Find the cross product of the vector \( \vec{v} = (4, 5, 6) \) and \( \vec{w} = (2, 3, 4) \).

More facts about the cross product

a. 

\[ \text{where } \theta \text{ is the angle between vectors } \vec{v} \text{ and } \vec{w} \]

**Therefore**, if \( \vec{v} \) and \( \vec{w} \) are parallel, then \( \vec{v} \times \vec{w} = \vec{0} \).

b. If \( \vec{v} \times \vec{w} \neq \vec{0} \), then the direction of \( \vec{v} \times \vec{w} \) is determined by the **right-hand rule**. Start at \( \vec{v} \), curl the fingers of your right hand towards \( \vec{w} \). You thumb will point to the direction of \( \vec{v} \times \vec{w} \).

Also can be thought of using the **wood screw rule**.
c. $|\vec{v} \times \vec{w}|$ is the area of the parallelogram spanned by the vectors $\vec{v}$ and $\vec{w}$

**Example:** Compute the area of the triangle with corners at $P = (0,1,1)$, $Q = (-1,0,2)$, and $R = (3,1,0)$.

**Example:** Find the equation of the plane containing the points $P = (0,1,1)$, $Q = (-1,0,2)$, and $R = (3,1,0)$.

One can think of $\vec{v} \times \vec{w}$ as a **torque** if you apply a force $\vec{w}$ on a rigid bar given by vector $\vec{v}$.

Section 12.9 problems #1-19 odd, 20, 32