Problem Session – April 12, 2006
Turn in tomorrow, April 13, 2006 at noon

Answer the following on a separate sheet of paper.

Use Green’s Theorem to compute the line integral $\oint_{\gamma} P \, dx + Q \, dy$

a. $(P,Q) = (x - y, x + y)$ and $\gamma$ is the circle of radius 1 with center $(0,0)$ traversed counterclockwise.

b. $(P,Q) = (x,0)$ and $\gamma$ is the curve described by the polar equation $r = 2\sin(3\theta)$ $0 \leq \theta \leq \frac{\pi}{3}$.

c. $(P,Q) = (\sin x, x + y)$ and $\gamma$ is the rectangle with vertices $(3,0),(3,5),(-1,5),$ and $(-1,0)$ traversed in that order.

Recall, we can parametrize the curve $y = x^2$ on $(0,1)$ with the parametrization $x = t$, $y = t^2$ $0 \leq t \leq 1$. A 1-dimensional object requires 1 parameter. Therefore, a 2-dimensional object requires 2 parameters. Let $P$ be the plane $z = 12 - 4x - 3y$. We can let $x$ and $y$ both be parameters like the following:

$$x = u, \ y = v, \ z = 12 - 4u - 3v \quad -\infty < u < \infty, \ -\infty < v < \infty$$

If we were interested in only part of the plane, there would be restrictions on $u$ and $v$.

2. Describe the surfaces with the following parametrizations:

a. $x = u, \ y = v, \ z = u - v$ where $-\infty < u < \infty, \ -\infty < v < \infty$

b. $x = u, \ y = v, \ z = u - v$ where $-2 \leq u \leq 3, \ 1 \leq v \leq 4$ (give specific points)

c. $x = u^2 + v^2, \ y = u, \ z = v$ where $-\infty < u < \infty, \ -\infty < v < \infty$.

d. $x = u^2 + v^2, \ y = u, \ z = v$ where $0 \leq u < \infty, \ 0 \leq v < \infty$.

e. $x = u, \ y = -\sqrt{9 - u^2 - v^2}, \ z = v$ where $-3 \leq u \leq 3, \ -3 \leq v \leq 3$

f. $x = \cos u, \ y = \sin u z = v$ where $0 \leq v \leq 5, \ 0 \leq u \leq 2\pi$ (What is the relationship between $x$ and $y$?)

g. $x = 4 \sin u \cos v, \ y = 4 \sin u \sin v, \ z = 4 \cos u$ where $0 \leq u \leq \frac{\pi}{2}, \ \pi \leq v \leq 2\pi$. (Think spherical)

3. Give parametrizations for the following:

a. The part of the plane $2x + 9y + z = 400$ that lies above the rectangle with vertices $(0,0),(1,0),(1,2),(0,2)$.

b. The circular cylinder that intersects the $y-z$ plane in the circle centered at $(0,0,3)$ with radius 2 and is free in the $x$-direction.

c. The part of the sphere centered at the origin that lies in the octant given by $x \geq 0, \ y \leq 0, \ z \leq 0$.

Homework: Read Section 13.4 and try exercises 1, 3, 5, 11-19odd.