Texas Hold’em Problem

Rules for Texas Hold’em:

1. The dealer shuffles a single deck of 52 playing cards.
2. Each player is dealt two cards face down; these are called the hole or pocket cards. First bets are made.
3. The dealer discards the top card in the remaining deck (burn card), then flips the next three cards face up on the table. These cards are called the flop cards. They are communal cards that are combined with the each player’s two pocket cards to form the players’ poker hands. Second bets are made.
4. The dealer burns the next top card and flips one more card face up on the table; this is the turn or fourth street card. Players make the best five-card poker hand they can from the six cards available (four communal and two hole cards). Third bets are made.
5. The dealer burns one more card and places the final card face up on the table; this is the river or fifth street card. Players use any of the seven cards (five communal and two hole cards) to make the best possible five-card poker hand. Final bets are made.
6. The player who shows the best hand wins! Ties can happen, and players with equal hands share the winnings.

Players may also fold (stop playing for that hand) at any point rather than betting.

Ranking of Hands (Best to Worst)

- Royal Flush: an Ace-high straight of the same suit (Ace, King, Queen, Jack and Ten).
- Straight Flush: a straight of the same suit.
- Four of a Kind: four cards of the same rank.
- Full House: a three of a kind and a pair.
- Flush: five cards of the same suit.
- Straight: five cards of sequential rank; an Ace may be high or low.
- Three of a Kind: three cards of the same rank.
- Two Pairs: two cards of one rank and two cards of a second rank.
- One Pair: two cards of the same rank.
- High Card: the highest card in the hand when none of the other (above) hands are achieved.

Questions:

1. Suppose one person is dealt five cards from a single deck of 52 playing cards. How many combinations of five cards are possible? What is the probability of getting each of the hands listed above? What is the sum of these probabilities? Given the probabilities for each type of hand, does the ranking make sense?

2. Suppose one person is dealt seven cards from a single deck of 52 playing cards. How many combinations of five cards are possible? What is the probability of getting each of the hands listed above? Are the probabilities the various hands better or worse than in (1)? Why is this true?
3. Suppose the five communal cards are: Ace of Spades, Ace of Clubs, Ace of Diamonds, Two of Hearts and Five of Spades. What is the worst hand any player could have? What is the best hand? Can you find the associated probabilities of these hands?

4. The two cards in your hand are a King of Spades and a King of Hearts. What communal cards would give you the best hand? What communal cards would give you the worst hand? What would the communal cards have to be to ensure that you are the winner of the round? Can you find the associated probabilities of these hands?

5. Randomly draw five cards from a deck and lay them face up. These are the communal cards. Redo problem (3) using your randomly drawn cards.

6. Randomly draw two cards from a deck. These are your two pocket cards. Redo problem (4) using your randomly drawn cards.

7. Players can place bets any time after the receive their pocket cards. If you have one pair in hand with no communal cards yet played, should you bet or fold (withdraw from the hand)? What kind of cards would you want to see after the flop in order to keep betting? What about after the turn and the river? Can you recommend a strategy for betting or folding?

8. Does the number of players in the game affect any of the problems above, assuming that the cards are all dealt randomly?

9. When thinking about how to answer these questions, how can we use simulation (physical or computer-generated) to approximate answers? What kinds of analytical processes can we use (e.g., decision trees)? What advantages and disadvantages do these methods have for these kind of questions?

10. What other kinds of applications in science or engineering could you find for the kinds of ideas presented in this problem?

References:


http://www.learn-texas-holdem.com/texas-holdem-odds-probabilities.htm

http://www.wcrl.ars.usda.gov/cec/java/possible.htm