A Knight’s Tour

What is a knight’s tour?
Given a grid of any size, is it possible for a knight to land on every square just once and end where he started.

How does a knight move?
A knight moves by going two squares in one direction, then one square in a perpendicular direction. For example, if a knight started in square 1, then he could move to either of the two starred squares.

Your goal: Find an algorithm (or strategy) for finding a knight’s tour on various sized boards. On this website, you can practice the tours:
http://www.borderschess.org/KnightTour.htm

Suggestions:
1. Try finding a knight’s tour on some different boards, such as a 3 by 3, 5 by 5, 5 by 6. Look for some patterns. If a tour doesn’t exist, you should be able to give reasons why.
2. Are certain moves required in certain squares? Why?

Extensions:
1. Given a knight’s tour, extend it to a bigger size. For example, take an 8 by 8 tour (one does exist!) and extend it to a 12 by 12. Come up with strategies to do this!
2. What about a queen’s tour or any other chess piece?
3. How do these tours relate to the traveling salesmen problem?