Locating Rocks

Abstract

How can math be used to locate rock (granite, oil, or hard bedrock) under the earth’s surface? When you locate a spot, how do you know how deep you should dig? To investigate this problem, we will solve a simplified version of this problem using your math skills from algebra II and geometry.

We begin the problem with a simplified model of reflection seismology. Suppose an explosive charge is detonated at some point on the flat surface of the earth. Numerous signals then spread out from the explosive source and propagate through the subsurface. What do you think the speed of these signals depend on?

For our purposes, we will initially assume that the subsurface is homogeneous (why?). We will also assume that the subsurface is of constant depth, \( d \) and the velocity of the signal, \( v \) is constant. Below the surface layer is hard bedrock off of which the pressure wave is reflected to eventually arrive again at the surface (see Figure 1). We will also suppose the receiver, called a geophone, is located \( X \) units from the source that detects the reflected signal at time \( T \) after the detonation of the charge.

![Figure 1: Diagram of the problem.](image)

Some problems:

1. Write an expression for the velocity of the signal in terms of \( s \) and \( T \) (recalling that distance = rate \times time).

2. Write an expression for \( s \) in terms of the depth \( d \) and \( X \).

3. What variables do we have in real life and which variables are we looking for? (ie - In a real example, could we measure the value of \( X \) ? What about \( d \)?)
4. Combine the equations from questions 1 and 2 to find the equation of a hyperbola (recall that the general form of a hyperbola is \( \frac{x^2}{a} - \frac{y^2}{b} = 1 \)) where \( x \) and \( y \) are the two variables we can not measure at the beginning.

5. A reflected seismic signal is heard at a geophone 300 meters away from the source 3 seconds after detonation. What is the slowest possible velocity?

6. A reflected seismic signal is received 300 meters from the source after 3 seconds. Is it possible for the signal to be received at a geophone 360 meters from the source after 4 seconds?

7. Consider the path from the source to the geophone that is reflected off the hard bedrock at depth, \( d \). Show that the total length of the path is smallest when the angle that the incident ray makes with the horizontal line at depth \( d \) is equal to the angle that the reflected ray makes with the horizontal. Conclude that the triangle is the figure above is isosceles.

8. A reflected signal is received 91 meters from the source after 1.1 seconds. The reflected signal is received at a second geophone 200 meters from the source after 2.1 seconds. Find the depth of the stratum (to the nearest meter) and the velocity of propagation (to the nearest meter per second).

Some possible extension:

1. Can you find a solution given that a reflected signal is received \( X_1 \) meters from the source after \( t_1 \) seconds, and that same signal is received at a second geophone \( X_2 \) meters from the source after \( t_2 \) seconds. Find the depth of the stratum.

2. What would change if you didn’t assume a homogeneous subsurface?