Changes for the 1st and 2nd printings of *A Friendly Intro. To Analysis* Second Edition, by Witold A.J. Kosmala (Jan. 2006)

On p. xiii, 6 lines from the bottom, my Web address is now changed to www.mathsci.appstate.edu/~wak.
On p. xiii, add "Numerical Integration" as the last line.
On p. xv, in line 4 of paragraph 2, change "Este" to "Esty".
On p. 3, in the last line of the footnote, it should be "Section 1.6".
On p. 4, in the answer to Example 1.1.4, "$A \setminus B = 5$" should be "$A \setminus B = \{5\}$".
On p. 4, in line above Theorem 1.1.5, the word "or" should be "and".
On p. 8, in Example 1.2.2, in the definition of S_6, N should be changed to Z.
On p. 11, line above Remark 1.2.8, "properly" should be changed to "property".
On p. 31, in Exercise 7(f), change "$a_{n+1} = \left[1 - \frac{1}{(n+1)^2}\right]$" to "$a_{n+1} = \left[1 - \frac{1}{(n+1)^2}\right] a_n$".
On p. 43, line (A6) should be: "There exists an element in F, distinct from 0, which we denote … ."
On p. 47, in Exercise 2(b), change "d" to a "b".
On p. 51, line (b) of Corollary 1.8.6 should be: "$\left| |a| - |b| \right| \le |a - b|$".
On p. 53, in Exercise 21(a), change the last inequality to a multiplication.
On p. 75, insert a minus sign into next to the last line in the Proof of part (a).
On p. 77, last term in the next to the last line should be n times the square root instead of nth root.
On p. 78, in line 7 of Remark 2.2.5, "$\lim_{n\to\infty} \frac{\sin(k/n)}{k/n}$" should be changed to "$\lim_{n\to\infty} \frac{\sin(k/n)}{k/n} = 1$".
On p. 83, change the text that comes above Theorem 2.3.6 to the following.

Proof. In determining whether to consider $+\infty$ or $-\infty$, writing out a few terms or simply observing that one of the leading terms has a negative coefficient and the other leading coefficient is positive, suggests that the limit is $-\infty$. Let $M > 0$ be given. We want to find n^* so that for all $n \ge n^*$, we will have $a_n < -M$. But, solving $a_n < -M$ for n is not easy. To avoid this task, we need to bound a_n above by something that tends to $-\infty$. Hence, in this problem we need to make the numerator a larger negative expression, and the denominator a smaller positive expression. Although there are many different choices, let us write

$$-n^3 + 1 < -\frac{1}{2}n^3, \text{ for } n \ge 2, \quad \text{and} \quad n^2 - n - 5 > \frac{1}{2}n^2, \text{ for } n \ge 5.$$

Therefore, picking $n \ge 5$, since the numerator is negative, we write

$$a_n = \frac{-n^3 + 1}{n^2 - n - 5} < \frac{-\frac{1}{2}n^3}{\frac{1}{2}n^2} = -n.$$

But $-n \le -M$ yields $n \ge M$. Thus, if $n^* \ge \max\{5, M\}$, for all $n \ge n^*$, we have $a_n < -M$. ■

The preceding lengthy proof can be shortened as shown next. Hopefully, this "behind-the-scenes" proof provided insight.

Pick any $M > 0$. Let $n^* \ge \max\{5, M\}$. If $n \ge n^*$, we have

$$a_n = \frac{-n^3 + 1}{n^2 - n - 5} < \frac{-\frac{1}{2}n^3}{\frac{1}{2}n^2} = -n \le -M .$$

Hence, $\lim_{n\to\infty} a_n = -\infty$. It should be noted that perhaps showing that $\{-a_n\}$ tends to $+\infty$ and implementing part (d) of Theorem 2.3.3 would be an easier approach. Moreover, since $a_n < -n$, using the comparison test would also prove the divergence to $-\infty$.

See Exercises 3 and 9 for more information concerning rational expressions. There are other ways to determine divergence to infinity. The next result relates ideas from previous sections to the divergence to infinity.

On p. 84, change part (c) of Theorem 2.3.7 to "If $\alpha = 1$, then $\{a_n\}$ may converge, diverge to plus or minus infinity, or oscillate."
On p. 85, in top line, change "three" to "four".
On p. 85, second line of the proof just below Example 2.3.8, change "$\lim_{n\to\infty} \frac{(n+1)^p}{b^n+1}$" to "$\lim_{n\to\infty} \frac{(n+1)^p}{b^{n+1}}$".
On p. 86, Exercise 8, change "diverges to infinity" to "diverges to plus or minus infinity".
On p. 87, Exercise 9, change "diverges to infinity" to "diverges to plus or minus infinity".
On p. 87, in Exercise 15 "three" to "four".
On p. 87, Exercise 16, change "diverge to infinity" to "diverge to plus or minus infinity".
On p. 89, line (d) of Definition 2.4.1, should have "$a_n < a_m$" instead of "$a_n \le a_m$".
On p. 103, Exercise 1, delete the first sentence. Start the problem with "Prove that …".
On p. 109, 5th line from the top change the sentences "The existence… Section 2.5. Why?" to "In Exercise 11 we are asked to show that there is a subsequence of $\{a_n\}$ that converges to s_0."
On p. 110, in Exercise 2(c), change "$r \ge 0$" to "$r \le -1$ or $r > 1$".
On p. 110, Exercise 5 should read as follows: Prove that every unbounded above sequence contains a monotone subsequence that diverges to plus infinity.
On p. 111 add Exercise 11 which states: "Complete the proof of Theorem 2.6.4."
On p. 120, part (b) should read as "f must be eventually bounded … ."
On p. 122, in Remark 3.1.12, change both p to n where $n \in N$.
On p. 125, in Exercise 14, prove the given two limits without using Theorem 3.1.13(b).
On p. 126, in Figure 3.2.1, fill in the point $(a, f(a))$ and make the vertical line above $a+\delta$ dotted.
On p. 129, in line right above Example 3.2.10, change "Theorem 3.2.5" to "Theorem 3.2.6".
On p. 168, Exercise 17 should read as "Give an example of a function f that is a continuous injection… ."
On p. 171, top line, change "what" to "that".
On p. 174, Exercise 3 should be changed to:

(a) Prove Theorem 4.4.7.
(b) Suppose $f:(a,b) \to \Re$ is continuous. Prove that if $f(a^+)$ and $f(b^-)$ are both finite, then f is bounded on (a,b). Explain why the converse is not true.
(c) Prove Corollary 4.4.8.
(d) Use Corollary 4.4.8 to prove that $f(x) = \sin\frac{1}{x}$ is not uniformly continuous on $(0,1)$ but $g(x) = x\sin\frac{1}{x}$ is uniformly continuous on $(0,1)$.

On p. 184, in Definition 5.1.1, second line, replace "$a \in D$" by "f is continuous at a".
On p. 191, parts (a) and (b) of Exercise should be changed to:

(a) continuous at exactly one point and differentiable at exactly one point.
(b) continuous at exactly two points and differentiable at exactly two points.

On p. 197, in the second line of the proof, delete "Thus, $f'(x) > 0$ or $f'(x) < 0$ on I^0".
On p. 197, next to the last line a derivative symbol is missing on f^{-1}.
On p. 198, in line 7 of the proof of Theorem 5.2.9, change "$pq > 0$" to "$p, q > 0$".
On p. 200, Exercise 7 should read as follows: "Give an example of a function f that is differentiable at $x = a$ such that $f'(a) \ne 0$, but yet f attains a relative extremum at $x = a$."
On p. 200, Exercise 8 should read as follows: "Give an example of a function f that is continuous at $x = a$, not differentiable at $x = a$, but yet f attains a relative extremum at $x = a$."
On p. 205, second indented equation has equal sign missing.
On p. 215, in the first line after the proof of Taylor's theorem, add a word " often" before the word "become."
On p. 247, the last line of the proof of Theorem 6.2.1 should be "Since $L(P,f)$ and $L(P,f)+\varepsilon$ are within ε of each other, so must be the upper and the lower integrals, proving the desired result."

On p. 257, keep the first 5 lines of the proof of Theorem 6.4.2. The rest of the proof should be changed to what follows.

This is a Riemann sum and thus, it follows that $L(P,f') \le f(b) - f(a) \le U(P,f')$. Since P is an arbitrary partition, we have that

$$\underline{\int_a^b} f' \le f(b) - f(a) \le \overline{\int_a^b} f'.$$

Lastly, since f' is Riemann integrable on $[a,b]$, upper and lower integrals must be equal and hence, $\int_a^b f' = f(b) - f(a)$.

On p. 261, Exercise 1, add at the end of the line "with $a > 0$".
On p. 295, change the top of page to what follows.

For any sequence $\{a_n\}\big|_{n=p}^{\infty}$, we can define a related sequence , $\{S_n\}\big|_{n=p}^{\infty}$ where

$$S_p = a_p$$
$$S_{p+1} = a_p + a_{p+1}$$
$$S_{p+2} = a_p + a_{p+1} + a_{p+2}$$
$$\vdots$$
$$S_n = a_p + a_{p+1} + a_{p+2} + \cdots + a_n = \sum_{k=p}^{n} a_k,\ p \le n.$$

Thus, S_n is the sum up to the term a_n. The sequence $\{S_n\}\big|_{n=p}^{\infty}$ is called the *sequence of partial sums* of the series $\sum_{k=p}^{\infty} a_k$. (See Exercise 15 of Section 2.2.) Subscripts are *dummy variables*. …

On p. 295, in Definition 7.1.2 and in Remark 7.1.3, change all $\{S_n\}$ to $\{S_n\}\big|_{n=p}^{\infty}$.
On p. 296, in the "Answer" 4 lines from the bottom, remove the first equality sign.
On p. 305, in Theorem 7.2.4(b), change "$\le$" to "$\ge$".
On p. 305, in the third line of the proof of part (a) of Theorem 7.2.4, this is the indented equation, $|a_k|$ is missing before $\le$ sign.
On p. 307, in part (b) of Remark 7.2.8, in the first line change $\ge$ to $>$.
On p. 313, in part (e), change "both ratio tests" to "Theorem 7.3.3 and Corollary 7.3.5".
On p. 323, Exercise 5 should start with three additional words "For each part,".
On p. 344, in the Proof of part (b), second sentence should be "Thus, choose a sequence $\{x_n\}$ in the interval $[0,1)$ that converges to 1, say, $x_n = \sqrt[n]{\frac{1}{2}}$."
On p. 350, in Exercise 2 add at the end "(Do not use Theorem 8.3.4.)"
On p. 350, in Exercise 4 add at the end "for the increasing case."
On p. 350, in Exercise 5(c), change "$f_n(x) \le f_{n+1}(x)$" to "$f_n(x) \le f_{n+1}(x)$ (or $f_n(x) \ge f_{n+1}(x)$)".
On p. 352, in the first line change "A series …" to "A converging series …".

On p. 358, in Exercise 11(e), an equal sign is missing.
On p. 362, in part (b) of Theorem 8.5.8, change $<$ to $\leq$.
On p. 383, heading for the Section 9.1 three lines from the bottom should have $\Re^3$ instead of $\Re^2$.
On p. 386, in first line, change to $\vec{k} = \langle 0,0,1 \rangle$.
On p. 387, capitalize the first word in the last paragraph.
On p. 404, in first line, change $\overrightarrow{P_0 P}$ to $\overrightarrow{P_0 P_1}$.
On p. 404, in indented line 6, change $-8x + 13y + 3k$ to $-8x + 13y + 3z$.
On p. 406, in tenth line from the bottom, change $-L_1$ to $= L_1$.
On p. 408, in 12th line from the bottom, add "$\vec{r}(t)$" between the words "if" and "represents".
On p. 416, in the 5th line from the bottom, there should be "$\vec{r}'(t)$" inside the integral instead of "$\vec{r}(t)$".
On p. 417, in the 3rd line from the top, there should be "$\vec{r}'(t)$" inside the integral instead of "$\vec{r}(t)$".
On p. 418, last line before Example 9.7.5, change "See Exercise 15." to "See Exercise 10 in Section 9.8."
On p. 449, 2nd line above Example 10.3.2, change "ration" to "ratio".
On p. 452, in the second line, change "very like" to "very much like".
On p. 454, in Exercise 1(a), change "top of a sphere" to "top half of a sphere".
On p. 459, in the first line of the proof of Theorem 10.4.5, change "we need" to "it is sufficient".
On p. 472, last line of the footnote should be: "See Part 3 of Section 12.8 in … ."
On p. 479, add the following paragraph on top of page.

It should be noted that finding all functions $f(x)$ for which $f(x) = f'(x)$ boils down to solving separable differential equation $\frac{dy}{dx} = y$. This was the content of Exercise 31(a) in Section 5.3.

On p. 536, in Exercise 10, change "part (f)" to "part (c)".
On p. 537, answer to Exercise 15 of Section 2.3 should be "$a_n = \frac{1}{n}$; $a_n = n$; $a_n = -n$; $a_n = (-1)^n n$".
On p. 543, answer to Exercise 20(b) in Section 5.4 should be actually 20(c). Corrected answer is
"$p_n(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^{n/2} \frac{x^n}{n!}$, $n = 0, 2, 4, \ldots$;".

On p. 545, Sec. 6.2, answer to Exercise 1 is actually an answer to Exercise 2.
On p. 546, answer to Exercise 7(d) of Section 6.4 should be "$x \arctan\frac{1}{x} + \frac{1}{2}\ln\left(x^2+1\right) + C$".
On p. 546, answer to Exercise 7(h) of Section 6.4 should be "$x - \frac{1}{2}\ln\left(x^2+2x+5\right) - \frac{1}{2}\arctan\frac{1}{2}(x+1) + C$".
On p. 546, answer to Exercise 7(m) of Section 6.4 should be "≈ 1.09".
On p. 547, answer to Exercise 15 of Section 6.5 should have "$f : [0,\infty) \to \Re$" in it.
On p. 549, answer to Exercise 8 of Section 7.4 given, is actually an answer to Exercise 8(c).
On p. 551, answer to Exercise 2(d) of Section 8.5 should be
"$p(x) = 3 - (x-1) + 2(x-1)^2 + (x-1)^3 + 0(x-1)^4 + \cdots$".

On p. 552, answer to Exercise 4 in Section 9.1 should be "$2x - 2y - 14z = -23$".
On p. 555, answer to Section 10.3 Exercise 1(a) should read that both partials do not exist.
On p. 556, answer to Exercise 4 of Section 10.6 given, is actually an answer to Exercise 3.
On p. 568, "functional values" is misspelled.