<u>In Depth Mathematics</u>

Taylor worked with the mean curvature in mathematics.

Characteristic of soap bubbles is that there is a constant mean curvature. Mean curvature can best be explained in taking any arbitrary object and picking a point on the surface of that object. Then take a plane that runs through the point and rotates around the normal vector. (The normal vector is found by taking the second derivative at that point.) Next, take average of all the curves formed by rotating the plane. This is called the mean curvature. An analytical way to find mean curvature is best explained by the following which is found in <u>Differential Geometry of Curves and Surfaces</u>:

The formula for finding the line of curvature is

$$K=(eg-f^2)/(Eg-F^2)=\{-\Psi'(\Psi'\phi''-\Psi''\phi')\}/\phi$$

Where ϕ is always positive, and $\Psi'=0$ where the tangent line to the generator curve is perpendicular to the axis or rotation (normal vector) or $\phi'\Psi''-\Psi'\phi''=0$ where the curvature of the generator curve is zero. We can simplify this by taking a derivative and get

$$K=-\phi''/\phi$$

This can be used to determine surfaces of revolution of curvature. To compute principal curvatures we can make the observation that:

$$K=eg/EG, \quad H=.5(eG+gE)/(EG)$$

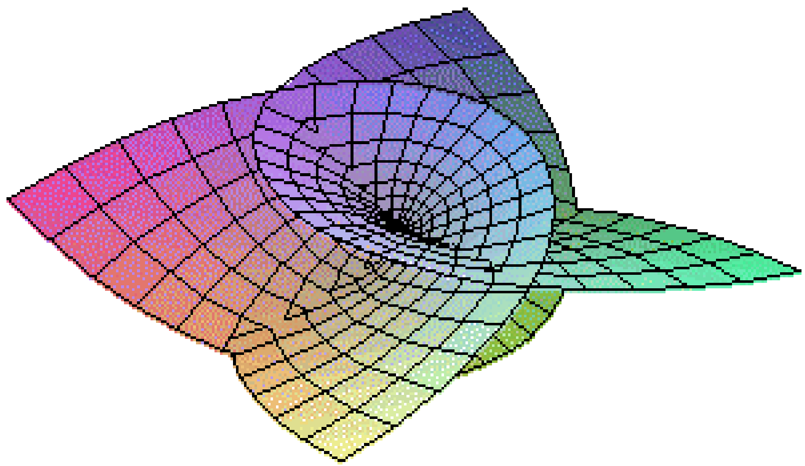
Thus the principal curvature of a surface of revolution is given by:

$$e/E=-(\Psi'\phi)/\phi^2=-\Psi'/\phi$$

$$g/G=\Psi'\phi''-\Psi''\phi'$$

Therefore, the mean curvature is:

$$H=.5(-\Psi'+\phi(\Psi'\phi''-\Psi''\phi')/\phi)$$



The equation to the above is:

$$\mathbf{X}(u,v)=(u-(u^3/3)+uv^2,\ v-(v^3/3)+uv^2,\ u^2-v^2)$$

From <u>Differential Geometry of Curves and Surfaces</u>, we are given that the largest curvature of the above surface is

$$k\_1=2/((1+u^2+v^2)^2)$$

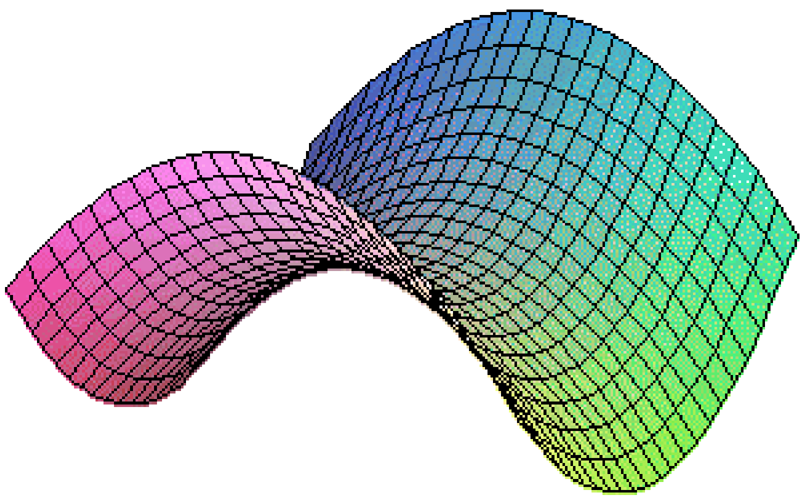
And the smallest curvature is given by the equation

$$K\_2=-2/((1+u^2+v^2)^2)$$

By this we can take the average of the two and get that the mean curvature is zero. This makes sense because if you pick ant point on the surface and rotate a plane about the normal vector, you will get a positive curvature and a negative curvature. The average of the two will be zero.

Let's look at another example that is also given in the same reference. Let's look at the following parametric equation and its graph:

$$\mathbf{x}(u,v)=(u,v,u^2-v^2) \text{ where } u=x,\ v=y.$$



From the above we get:

$$h(u,v)=u^2-v^2$$

Now we need to take the partial derivatives of h(u,v)

$h_u=2u$

$h_v=-2v$

$h_{uu}=2$

$h_{vv}=-2$

$h_{uv}=h_{vu}=0$

The equation that is given for mean curvature is:

$$H=\frac{[(1+h_x^2)*h_{yy}-2h_x*h_y*h_{xy}+(1+h_y^2)h_{xx}]}{2(1+h_x^2+h_y^2)^{(3/2)}}$$

If we take the values from the example above we get:

$$H=\frac{((1+(2u)^2)(-2)+(1+(-2v)^2)(2))}{(1+(2u)^2+(2v)^2)^{(3/2)}}$$

From this we can infer that the mean curvature will be zero.