Homework 2: Curves

You may work alone or in a group of up to 3 people and turn in one per group. You should be prepared to present any of the problems your group turned in. Each group must write up their work in their own words.

4140 students complete all but the last problem - graduate students in 5530 complete all of them. The maple files listed below are on the class webpage.

The purpose of homework is to learn and practice computational strategies, concepts, and develop critical thinking and problem-solving skills, so you should try problems on your own. Feel free to talk to me or each other if you are stuck on this assignment, but be sure to acknowledge any sources outside your group, including each other, like “The insight for this solution came from a conversation with Joel.” If you know how to do a problem and are asked for help, try to give hints rather than the solution.

1. The sights of our everyday lives are filled with curves, many recognizable. You’ll have to open your eyes for this problem. Find, either in person or in a picture, a curve which strikes a chord with you. (Circles and lines are unacceptable, be more creative.) Describe where the curve arises or is located, and what significance it has. Why is it personally meaningful? Sketch it; can you parameterize it? Art, architecture, and nature are great places to search for resonating curves.

2. Choose your favorite planar curve. Write down a parametric version of the curve. Use the Maple Applet \texttt{spacecurve.mw} to find the Plot, Velocity, Acceleration, Jerk, Speed, ArcLength, Curvature, and Torsion, and write down what Maple finds. Next provide a formula for calculating each term by hand (equation and/or words) and explain in your own words what each of the pieces means physically and/or geometrically. An example explanation for velocity might be something like:

   \textit{The velocity is the first derivative of position componentwise (with respect to time) so if }r(t) = (x(t), y(t), z(t)) \textit{ then the velocity is } (x'(t), y'(t), z'(t)) \textit{ and it gives us a tangent to the curve at the given point. The velocity represents the way the position is changing as both a direction and a number, the speed, which is the length of the velocity vector. A tangent vector is the best fit line to a curve at a point.}

You may use a web search and/or the text to help you but be sure to give proper credit.

3. Choose your favorite non-planar curve. Explain how we know that the torsion will be non-zero. Write down the parametric version of the curve. Use the Maple Applet \texttt{spacecurve.mw} to find the Plot, Velocity, Acceleration, Jerk, Speed, ArcLength, Curvature, and Torsion, and write down what Maple finds.

4. Choose one of the following curves and enter it into the Maple Applet \texttt{TNBapplet.mw}:
   
   \begin{itemize}
   \item Witch of Agnesi \((2t, \frac{2}{1+t^2}, 0)\)
   \item OR Lemniscate of Bernoulli \((\frac{3\cos t}{1+\sin^2 t}, \frac{3\sin t\cos t}{1+\sin^2 t}, 0)\)
   \item OR Vivani’s Curve \((1 + \cos t, \sin t, 2\sin \frac{t}{2})\)
   \end{itemize}

   Enter into Maple: \((2*t, 2/(1+t^2), 0)\) with \(t\) ranging from -1 to 1

   \textsc{OR} \((3\cos(t)/(1+\sin(t)*\sin(t)), 3\sin(t)\cos(t)/(1+\sin(t)\sin(t)), 0)\)

   with \(t\) ranging from \(-2\pi\) to \(2\pi\)

   \textsc{OR} \((1+\cos(t), \sin(t), 2\sin(t/2))\) with \(t\) ranging from \(-2\pi\) to \(2\pi\)

   \begin{itemize}
   \item (a) Sketch the curve and the Frenet Frame at two different places on the curve. Distinguish between T, N and B on your frame, either by color or by labels.
   \end{itemize}
(b) Next provide a formula for calculating each vector in the Frenet Frame (equation and/or words) and explain in your own words what each of the pieces means physically and/or geometrically.

(c) Search for information about the curve in our book and on the web and summarize what you found in your own words.

5. Cycloid \( \alpha(t) = (t + \sin t, 3 - \cos t, 0) \)
Enter the curve into the Maple Applet \texttt{TNBapplet.mw} as \((t+\sin(t), 3 - \cos(t),0)\) with \(t\) ranging from 0 to 7

(a) Sketch the curve and the Frenet Frame at two different places on the curve. Distinguish between T, N and B on your frame, either by color or by labels.

(b) Is the Frenet Frame defined everywhere in this domain? If not, specify any problem points, and explain whether any of the vector components of the Frenet Frame are defined at the problem points. For those vector components that are not defined, explain why not. Check your assertions by testing out various \(t\) values.

(c) Why is the cycloid interesting from a physics standpoint? Use a web search, but write this up in your own words.

6. Spiral \( \alpha(t) = (3 \cos t, 3 \sin t, \log t) \)
Enter the curve into the Maple Applet \texttt{TNBapplet.mw} as \((3*\cos(t), 3*\sin(t),\log(t))\) with \(t\) ranging from \(.0000001\) to \(2\pi\)

(a) Sketch the curve and the Frenet Frame at two different places on the curve. Distinguish between T, N and B on your frame, either by color or by labels.

(b) Is the Frenet Frame defined everywhere in this domain? If not, specify any problem points, and explain whether any of the vector components of the Frenet Frame are defined at the problem points. For those vector components that are not defined, explain why not. Check your assertions by testing out various \(t\) values.

(c) Why is a helical curve interesting from a physics standpoint? Use a web search, but write this up in your own words.

7. Graduate Problem (Extra Credit for 4140 students): Computer Images of Curves

(a) In Maple, use a plot command to plot the planar curve \( y = \sqrt{10^{-30} + x^2} \). When plotted from \(x=-1..1\), the curve looks like it behaves the same as \( y = \|x\| \) at the origin. Test out smaller ranges of \(x\) in the form of \(\frac{1}{10^{..0}}\) (i.e. \(x = -\frac{1}{10}..\frac{1}{10}\), etc). Can you distinguish the behavior of these curves at the origin by a similar Maple plot command? If so, what is the largest value of \(x\) of the form \(x = -\frac{1}{10^{..0}}..\frac{1}{10^{..0}}\) that can distinguish them?

(b) Can you distinguish the behavior of the curves at the origin using differential geometry techniques? If so, explain how, and show work to distinguish the curves at the origin.