

9.1 Sequences in Maple

A *sequence of integers* is an ordered list of numbers, like the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21...

You have explored sequences before, in middle and high school. In the calc 2 setting we are interested in more general sequences and in understanding the limit as $n \rightarrow \infty$ —whether a sequence converges or diverges, among other concepts.

- If $a_n = \frac{n}{e^n}$, write out (but do not simplify) the first three terms of the sequence.
- Does L'Hôpital's apply here?
- What is $\lim_{n \rightarrow \infty} a_n$? Show reasoning or work.

Open Maple. When you first launch it, there are icons. After choosing the **calculus** icon, at the bottom of the list open the **sequences** applet. (If that doesn't work, you can open it from the class highlights page).

Be sure that Check for Convergence is checked

Examine the last four sequences in the pull down menu and take notes as you investigate:

1. $a_n = (2n + 1)^2$

- (a) The sequence _____ converges to _____ diverges
- (b) Sketch a rough plot of the sequence

(c) (By-hand) Substitute the following n into the sequence and write out the terms:

$n = 1$ _____ $n = 2$ _____ $n = 3$ _____

2. $a_n = \left(\frac{-1}{2}\right)^{n-1} - 1$

- (a) The sequence _____ converges to _____ diverges
- (b) Sketch a rough plot of the sequence

(c) (By-hand) Substitute the following n into the sequence and write out the terms:

$n = 1$ _____ $n = 2$ _____ $n = 3$ _____

3. $a_n = \sin\left(\frac{1}{n^2}\right) + \cos\left(\frac{1}{n}\right)$

(a) The sequence _____ converges to _____ diverges

(b) Sketch a rough plot of the sequence

4. $a_n = \frac{\cos(n)}{n}$

(a) The sequence _____ converges to _____ diverges

(b) Sketch a rough plot of the sequence

5. Which, if any of the sequences always increases ($a_n \leq a_{n+1}$ for all n)?

$a_n = (2n + 1)^2$ $a_n = \left(\frac{-1}{2}\right)^{n-1} - 1$ $a_n = \sin\left(\frac{1}{n^2}\right) + \cos\left(\frac{1}{n}\right)$ $a_n = \frac{\cos(n)}{n}$

6. Which, if any of the sequences always decreases ($a_n \geq a_{n+1}$ for all n)?

$a_n = (2n + 1)^2$ $a_n = \left(\frac{-1}{2}\right)^{n-1} - 1$ $a_n = \sin\left(\frac{1}{n^2}\right) + \cos\left(\frac{1}{n}\right)$ $a_n = \frac{\cos(n)}{n}$

7. A monotone sequence is one that always increases or always decreases, so circle the monotone sequences from your last two responses:

$a_n = (2n + 1)^2$ $a_n = \left(\frac{-1}{2}\right)^{n-1} - 1$ $a_n = \sin\left(\frac{1}{n^2}\right) + \cos\left(\frac{1}{n}\right)$ $a_n = \frac{\cos(n)}{n}$

8. Are any of the sequences bounded (ie the a_n terms stay within a fixed region as $n \rightarrow \infty$, rather than approaching $\pm\infty$)? You can increase the number of terms in the sequence in the Maple applet.

$a_n = (2n + 1)^2$ $a_n = \left(\frac{-1}{2}\right)^{n-1} - 1$ $a_n = \sin\left(\frac{1}{n^2}\right) + \cos\left(\frac{1}{n}\right)$ $a_n = \frac{\cos(n)}{n}$ none

“A sequence per-se is inherently and literally primordial, and for me, this is a very alluring creative outlet. In other words, to be inventive in this medium, one must have mastery of only two things, numbers and ideas.”

[Neil Sloane, founder of the Online Encyclopedia of Integer Sequences]