9.1 Sequences in Maple

A sequence of integers is an ordered list of numbers, like the
Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21...
You have explored sequences before, in middle and high school. In the calc 2 setting we are interested in the limit as $n \to \infty$—whether a sequence converges or diverges, among other concepts.

Open Maple. When you first launch it, there are icons. After choosing the calculus icon, at the bottom of the list open the sequence applet. (If that doesn't work, you can open it from the class highlights page).
Be sure that *Check for Convergence* is checked

Examine the last four sequences in the pull down menu and take notes as you investigate:

1. $a_n = (2n+1)^2$

   (a) The sequence converges to _______________ diverges

   (b) Sketch a rough plot of the sequence

   (c) (By-hand) Substitute the following n into the sequence and write out the terms:

   $n = 1$ _______________ $n = 2$ _______________ $n = 3$ _______________

2. $a_n = (\frac{-1}{2})^{n-1} - 1$

   (a) The sequence converges to _______________ diverges

   (b) Sketch a rough plot of the sequence

   (c) (By-hand) Substitute the following n into the sequence and write out the terms:

   $n = 1$ _______________ $n = 2$ _______________ $n = 3$ _______________

3. $a_n = \sin(\frac{1}{n^2}) + \cos(\frac{1}{n})$

   (a) The sequence converges to _______________ diverges

(b) Sketch a rough plot of the sequence

4. $a_n = \dfrac{\cos(n)}{n}$

   (a) The sequence   converges to ______________   diverges

   (b) Sketch a rough plot of the sequence

5. Which, if any of the sequences always increases ($a_n \leq a_{n+1}$ for all n)?

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\dfrac{1}{n^2}) + \cos(\dfrac{1}{n})$ $a_n = \dfrac{\cos(n)}{n}$

6. Which, if any of the sequences always decreases ($a_n \geq a_{n+1}$ for all n)?

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\dfrac{1}{n^2}) + \cos(\dfrac{1}{n})$ $a_n = \dfrac{\cos(n)}{n}$

7. A monotone sequence is one that always increases or always decreases, so circle the monotone sequences from your last two responses:

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\dfrac{1}{n^2}) + \cos(\dfrac{1}{n})$ $a_n = \dfrac{\cos(n)}{n}$

8. Are any of the sequences bounded (ie the a_n terms stay within a fixed region as $n \to \infty$, rather than approaching $\pm\infty$)? You can increase the number of terms in the sequence in the Maple applet.

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\dfrac{1}{n^2}) + \cos(\dfrac{1}{n})$ $a_n = \dfrac{\cos(n)}{n}$ none

9. Do any of the sequences alternate from positive to negative, from one term to the next? (alternating sequence)

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\dfrac{1}{n^2}) + \cos(\dfrac{1}{n})$ $a_n = \dfrac{\cos(n)}{n}$ none

"A sequence per-se is inherently and literally primordial, and for me, this is a very alluring creative outlet. In other words, to be inventive in this medium, one must have mastery of only two things, numbers and ideas."
[Neil Sloane, founder of the Online Encyclopedia of Integer Sequences]