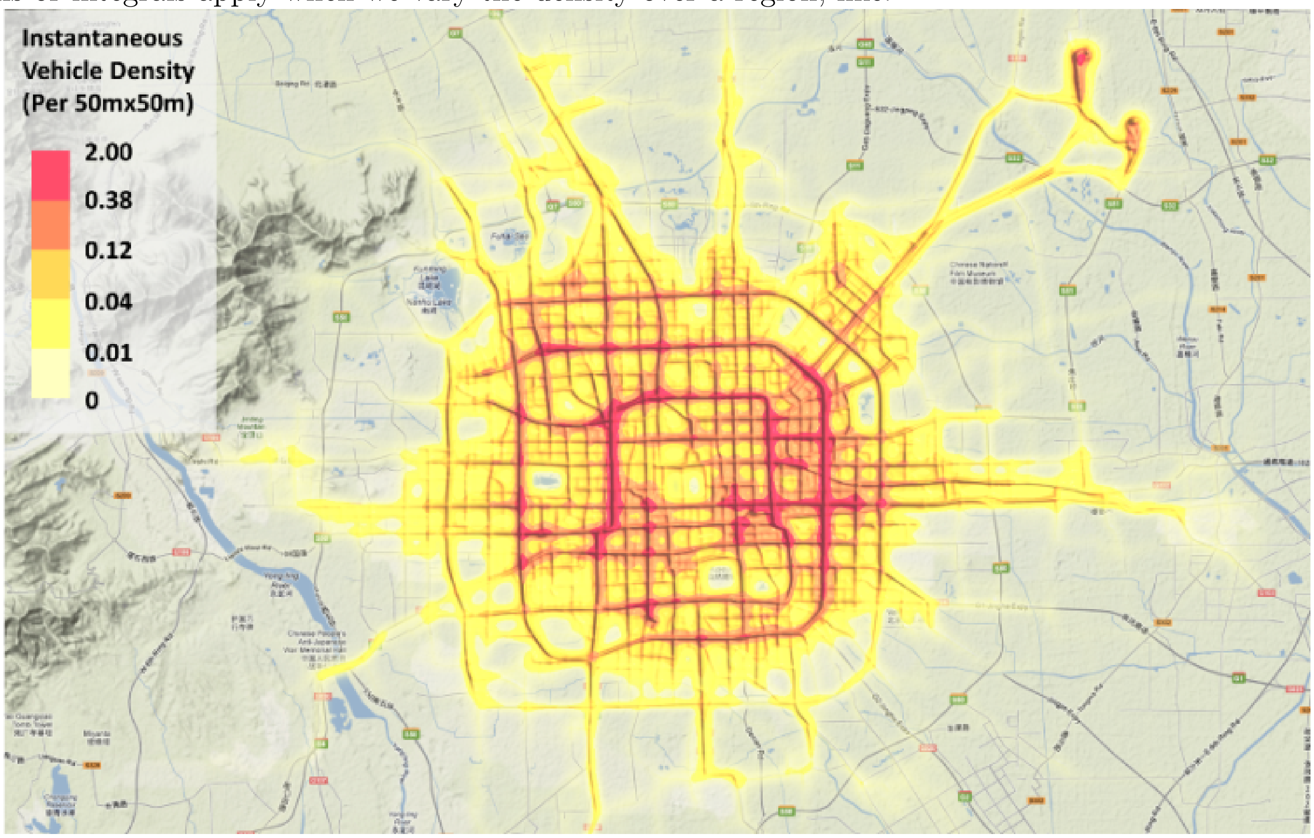
8.4 Density Applications of 8.1 and 8.2

Read through the following and **answer questions on a piece of paper**.

The mass of a substance or a population count is often computed as a density times a length, area, or volume. However, this only applies if the density is constant over the region, like you would have learned about starting in middle grades science classes.
However, there are many scenarios where the density is not a constant. Riemann sum approximations or integrals apply when we vary the density over a region, like:



by Zuchao Wang, Xiaoru Yuan, who note that: "Traffic density rendering is important, because it gives an intuitive overview of massive trajectories. It is also related to the study of traffic jams, hot spots and people's behaviors. We use density map algorithm to generate pictures of traffic density in Beijing, with real taxi GPS data." (http://vis.pku.edu.cn/trajectoryvis/en/densitymap.html)

In Calculus II, we will vary the density in one direction (Calculus III would handle more complex situations). The idea is to slice so that the density is approximately constant on a slice.

1. As a simplified example of traffic density, suppose that the function $\delta(x) = 200 + 100e^{-.1x}$ models the density of traffic on a straight road, measured in cars per mile, where x is the number of miles east of a major interchange.

   (a) What are the units on $\delta(x)\Delta x$, where Δx is a small slice of the road?

   (b) We'll add up the different densities on each slice and form the Riemann sum $\sum \delta(x)\Delta x = \sum 200 + 100e^{-.1x} \quad \Delta x$ and then the integral $\int_0^2 200 + 100e^{-.1x} \quad dx$
   Evaluate the integral using w-subs.

   (c) Write a sentence to explain the meaning of the value you found in this real-life context.

2. Consider the cone that has a base of radius 4 m and a height of 5 m.

   (a) Picture the cone lying horizontally with the center of its base at the origin and think of the cone as a solid of revolution. Sketch and label it.

   (b) Use this to write and evaluate a definite integral whose value is the volume of the cone. Show reasoning/work and additional sketches to solve for any needed lengths.

   (c) Next, suppose that the cone has uniform density of 800 kg/m^3. What is the mass of the solid cone?

   (d) Now suppose that the cone's density is not uniform, but rather that the cone is most dense at its base. In particular, assume that the density of the cone is uniform across cross sections parallel to its base, but that in each such cross section that is a distance x units from the origin, the density of the cross section is given by the function $\delta(x) = 400 + \frac{200}{x^2}$, measured in kg/m^3. First think about the mass of a given slice of the cone x units away from the base; remember that in such a slice, the density will be essentially constant.

   (e) Now add up all the slices in a Riemann sum.

   (f) Then determine and evaluate a definite integral whose value is the mass of this cone of non-uniform density.

3. Group Work:

   Appalachian's General Education Program prepares students to employ various modes of communication. Successful communicators interact effectively with people of both similar and different experiences and values.

   (a) First, swap papers with a neighbor or two.

   (b) Look through your neighbor's paper and circle anything that differed from your paper or that you have a question on.

   (c) Next, hand back the paper.

   (d) Discuss.

Portions of Questions 1 and 2 were adapted from *Active Calculus*.