8.5 Work: Varying Force

- Work is force $\times$ distance displaced only applies if the force is constant while it is exerted over the distance
- Integrals apply when we vary the force. The idea is to slice so that the force is approximately constant on a slice for its displacement. Then $\sum \to \int$ like
  Hook's Law to stretch (and hold) a spring, where $F(x) = kx$ constant for displacement Δx and $W = \int F(x)dx$
- Sometimes need to calculate the force, like when it is a column of water: mass = density $\times$ volume, F = mass $\times$ g
- Often we won't need to multiply by g like when we have a density that already has a force component:
  weight (force in lbs) = volume of a slice $\times$62.4 lbs/ft^3
  work on a slice = volume $\times$62.4 lbs/ft$^3\times$ slice displacement

Clicker Question

1. If we have a cylindrical oil tank of radius 3 m and height 10 m standing up on its circular base like a garbage can would (ie NOT sideways), where h is the height from the bottom of the tank, then what is the work to pump out the oil, assuming oil has a density of $800kg/m^3$?

a) $\int_0^{10} 2\pi h\delta(h)\, dh$

b) $\int_0^{10} 800 \times \pi 3^2\, dh$

c) $\int_0^{10} 800 \times 2\sqrt{3^2 - h^2} \times dh \times (10 - h)$

d) other

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F d = ($800 kg/m^3 \times$ volume) $\times$ distance the slice displaced

$\int_0^{10} 800 \times \pi 3^2\, dh (10 - h)$

History and Applications

- Archimedes buoyant forces inherent in fluids
- Sir Isaac Newton
- Work = weight lifted through a height: 1826 French mathematician Gaspard-Gustave Coriolis
  steam engines water out of flooded ore mines