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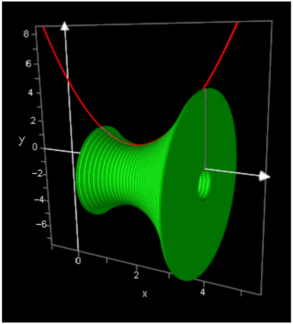
  annular/washer region:



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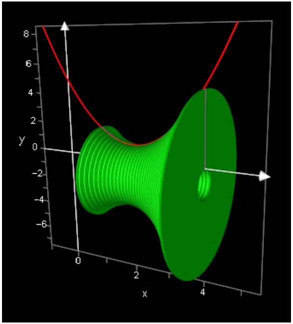

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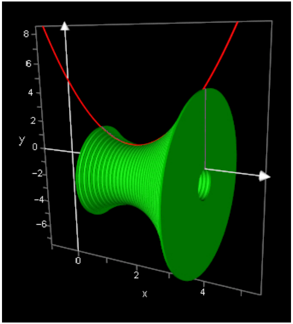

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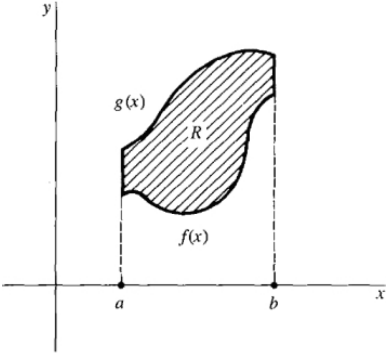
- Key is to figure out the radius (or radii) via pics

What I want you to show me... reasoning for radius, integral

Clicker Question

1. If R is rotated about the $x-$axis then the volume is given by



a) $\int_a^b (g(x) - f(x))\, dx$
b) $\int_a^b \pi (g(x)^2 - f(x)^2)\, dx$
c) both of the above
d) none of the above
e) no way to tell without more information

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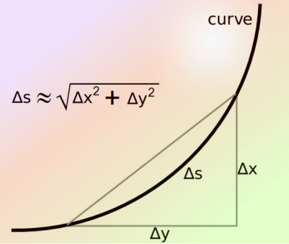
- Pythagorean looks good until we see BOTH $\triangle x, \triangle y$



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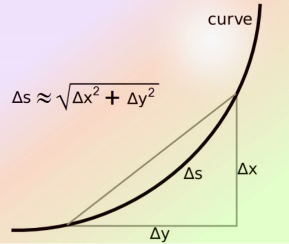

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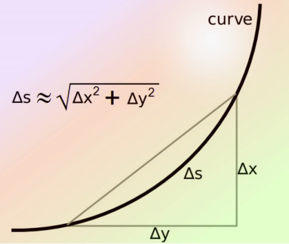

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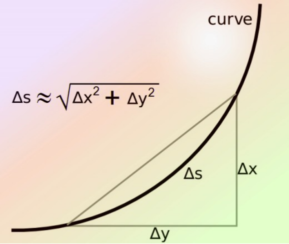

- $\frac{\triangle y}{\triangle x} \approx$ slope $= f'(x)$, so $\triangle y \approx f'(x)\triangle x$
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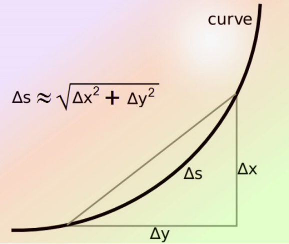

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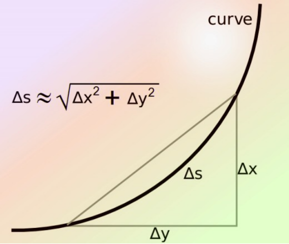

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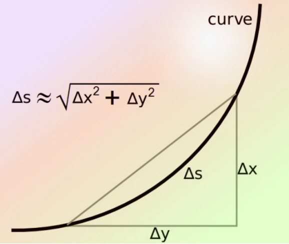

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- arc length $= \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

What I want you to show me... f', integral

Clicker Question

2. The length of the graph of $y = \sin(x^2)$ from $x = 0$ to $x = 2\pi$ is calculated by

a) $\int_0^{2\pi} cos(x^2)dx$

b) $\int_0^{2\pi} \sqrt{1 + \sin x^2}dx$

c) $\int_0^{2\pi} \sqrt{1 + \cos^2(x^4)}dx$

d) $\int_0^{2\pi} \sqrt{1 + \cos^2(x^2)}dx$

e) none of the above

History and Applications

- Johannes Kepler (1571-1630) computed the volume of a torus
- 1641 Evangelista Torricelli: Torricelli's Trumpet
- length of an irregular arc was thought to be impossible to compute.
- approximating π
- logarithmic spiral (Torricelli/John Wallis), cycloid (Christopher Wren), catenary (Gottfried Leibniz)
- Hendrik van Heuraet and Pierre de Fermat
- *Arc-Length Parameterized Spline Curves for Real-Time Simulation... Motion control is simple if object trajectories are parameterized by arc length*