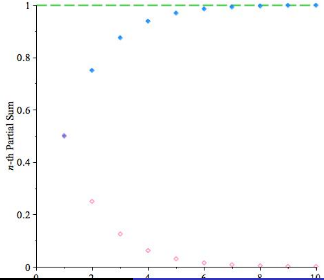
9.2 Geometric Series Review

- Geometric Series a= starting term, x=constant ratio of each term to preceding one
  $\sum_{i=0}^{\infty} ax^i = \frac{a}{1-x}$ when$|x| < 1$ and diverges otherwise

n^{th} partial sum (1st n terms added): $\sum_{i=0}^{n-1} ax^i = \frac{a(1-x^n)}{1-x}$ for $x \neq 1$

Example: $\sum_{i=0}^{n-1} \frac{1}{2}\frac{1}{2}^i = \sum_{i=1}^{n} \frac{1}{2}^i$ careful of starting # and index



9.3 Series: Partial Sums More Generally

- $\sum_{n=1}^{\infty} a_n$ and convergence? [9.3, 9.4, 9.5, chapter 10]

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- Example: $\sum\limits_{n=1}^{\infty} \frac{1}{n(n+1)}$

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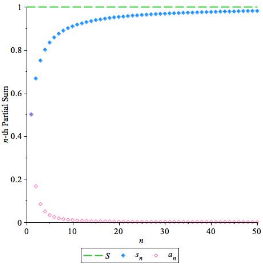
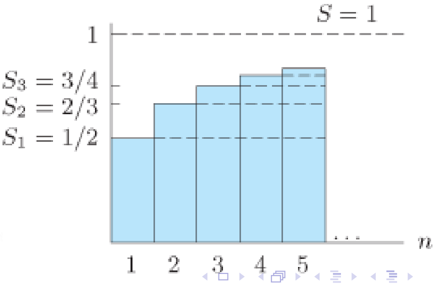
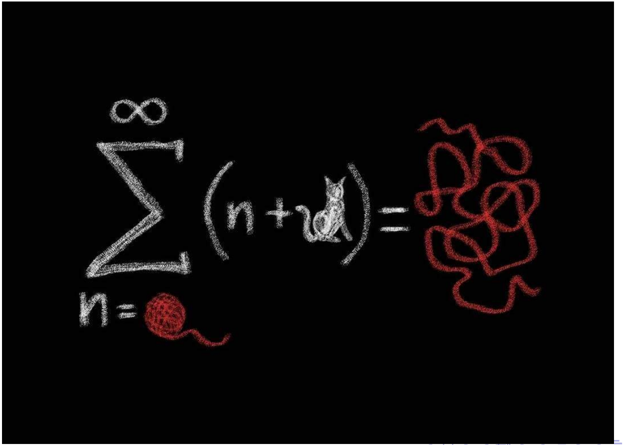
- Example: $\sum\limits_{n=1}^{\infty} \frac{1}{n(n+1)}$ $\quad S_n = \frac{n}{n+1}$ $\quad \lim\limits_{n\to\infty} S_n = 1$





9.3: Terms Not Going to 0

- terms not going to 0: $\lim_{n\to\infty} a_n \neq 0$ or DNE, then partial sums diverge and so does the series. Example: $\sum_{n=1}^{\infty} \frac{5+n}{2n+1}$



Clicker Question

1. What can we say about the series $\sum_{n=1}^{\infty}(1)^n$?

a) it is a geometric series with a constant ratio of each term to its preceding one x

b) we can find a pattern for the partial sums $S_n = \sum_{i=1}^{n} a_i$

c) $\lim_{n\to\infty} a_n \neq 0$ so we can apply the terms not going to 0

d) all of the above

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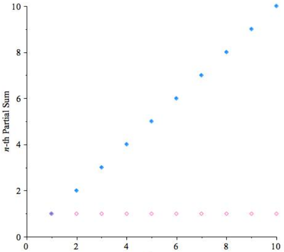

9.3:Linearity for Convergence or Divergence

- Linearity: $\sum_{n=1}^{\infty} a_n$ converges to S and $\sum_{n=1}^{\infty} b_n$ converges to T, and k is any constant, then $\sum_{n=1}^{\infty} ka_n + b_n$ converges to $kS + T$.

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  Application 1: add two geometric series (converge to sum)

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  Application 1: add two geometric series (converge to sum)
  Application 2: add divergent & convergent series (diverge)
  Example: $\sum_{n=1}^{\infty} (\frac{1}{2})^n + 1^n$.
  Diverges, because if it were convergent, then subtract convergent $\sum_{n=1}^{\infty} (\frac{1}{2})^n$ and the result should converge by linearity, but doesn't!

Clicker Question

2. What can we say about $\sum_{n=1}^{\infty} \frac{1}{3^n} + \frac{1}{2^n}$?

a) It is a geometric series so we can apply 9.2 methods to determine convergence by checking if $|x| < 1$ or divergence otherwise

b) We can use the $\lim_{n\to\infty} a_n \neq 0$ to determine divergence

c) We can use linearity to determine convergence or divergence

d) all of the above

e) none of the above

Clicker Question

2. What can we say about $\sum_{n=1}^{\infty} \frac{1}{3^n} + \frac{1}{2^n}$?

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c) We can use linearity to determine convergence or divergence

d) all of the above

e) none of the above

$$= \sum_{n=1}^{\infty} \frac{1^n}{3^n} + \frac{1^n}{2^n} = \sum_{n=1}^{\infty} (\frac{1}{3})^n + (\frac{1}{2})^n$$

Clicker Question

3. Do any of the following apply to $\sum_{n=1}^{\infty} \frac{1}{n}$?

a) It is a geometric series so we can apply 9.2 methods to determine convergence by checking if $|x| < 1$ or divergence otherwise

b) We can use the $\lim_{n\to\infty} \frac{1}{n} \neq 0$ to determine divergence of $\sum_{n=1}^{\infty} \frac{1}{n}$

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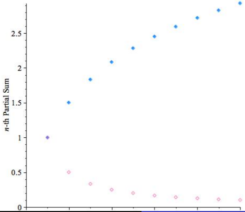

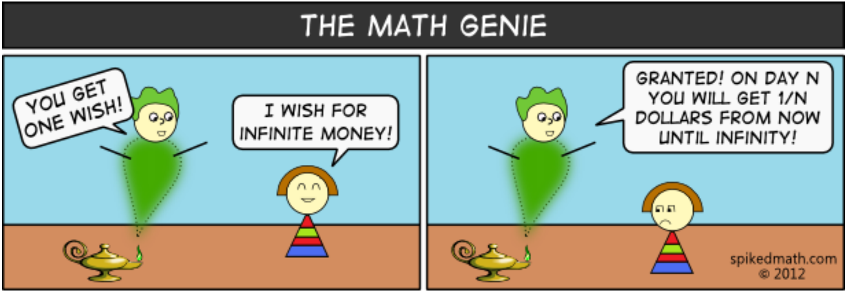
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I WISH FOR INFINITE MONEY!
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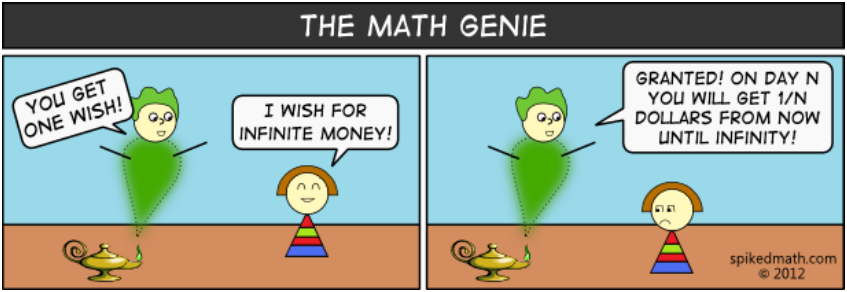
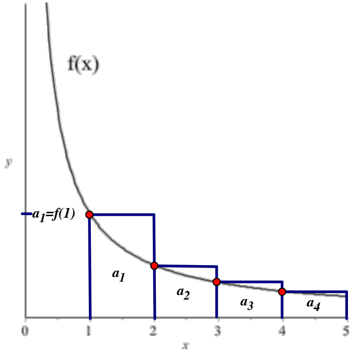


Harmonic series $\sum_{N=1}^{\infty} \frac{1}{N}$ diverges by growing to ∞ slowly! Why?

Integral Test

9.3: Integral Test Bounds

If series has terms that are decreasing and positive (eventually), the integral test not only tells us about convergence, but also bounds the series:



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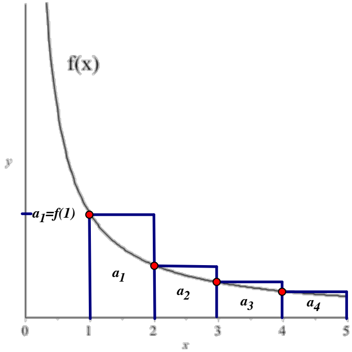

$$\int_1^{\infty} f(x)dx \leq \sum a_n$$

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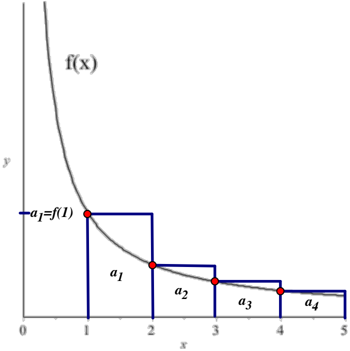

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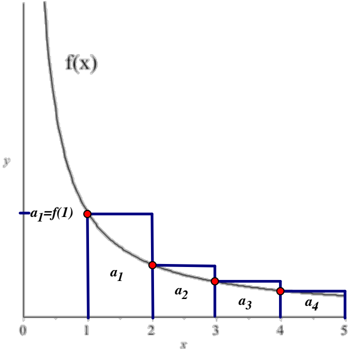
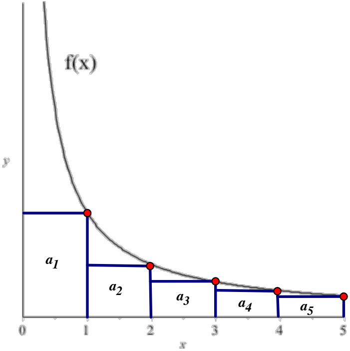
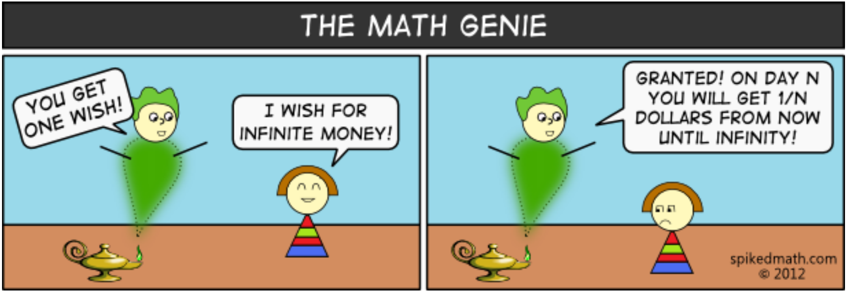




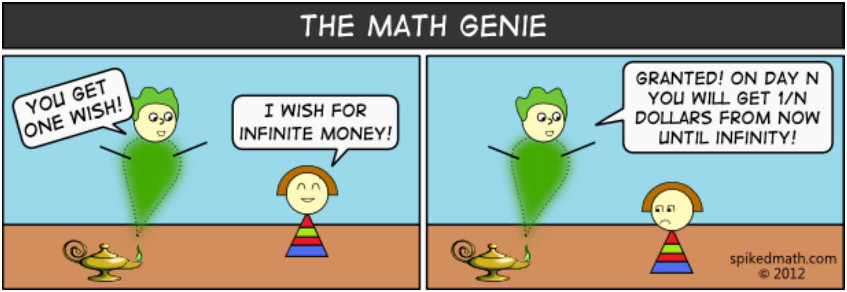
$$\int_1^\infty f(x)dx \leq \sum a_n \leq a_1 + \int_1^\infty f(x)dx$$



Harmonic series $\sum_{N=1}^{\infty} \frac{1}{N}$ diverges by growing to ∞ slowly! Why?

Integral Test

assumptions: terms are decreasing, $a_n > 0$, known integral



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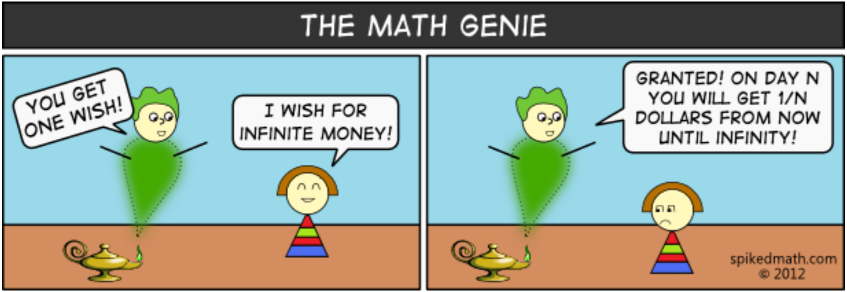

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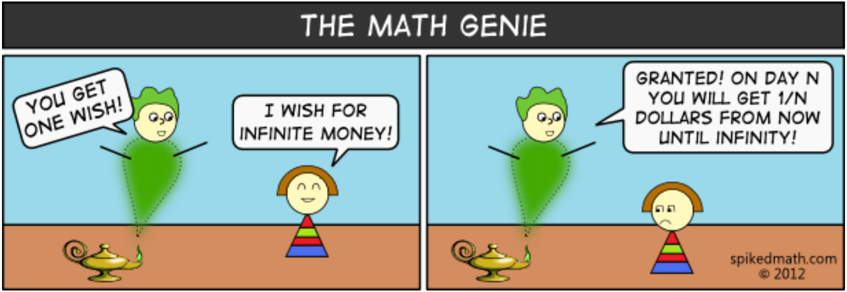
$$\int_1^{\infty} \frac{1}{x} dx = \lim_{b\to\infty} \int_1^{b} \frac{1}{x} dx =$$



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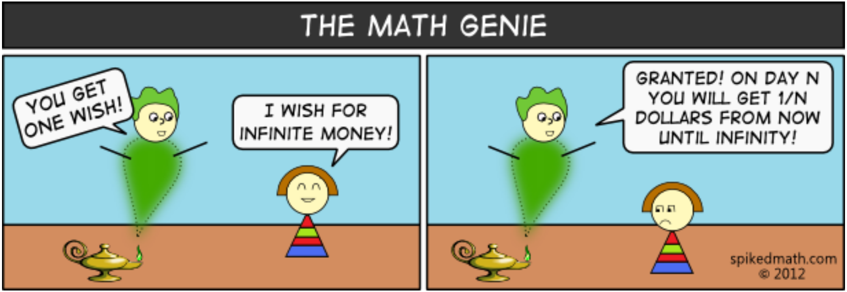
$$\int_1^\infty \frac{1}{x} dx = \lim_{b \to \infty} \int_1^b \frac{1}{x} dx = \lim_{b \to \infty} \ln(x)\Big|_1^b =$$



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Integral Test

assumptions: terms are decreasing, $a_n > 0$, known integral yes so test applies

$$\int_1^{\infty} \frac{1}{x} dx = \lim_{b\to\infty} \int_1^{b} \frac{1}{x} dx = \lim_{b\to\infty} \ln(x)\Big|_1^b = \lim_{b\to\infty} \ln(b) - \ln(1)$$

diverges so series does too

9.3: Integral Test

- For $\sum\limits_{1}^{\infty} a_n$, if the terms are decreasing and $a_n > 0$ then the series behaves the same way as $\int\limits_{1}^{\infty} a_n dn$.
  So look for decreasing and positive terms (eventually) that we can integrate (Calc I or Chap 7) + improper integral. Otherwise the test does NOT help.

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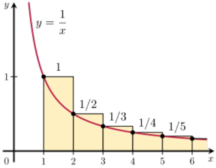
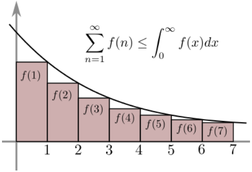
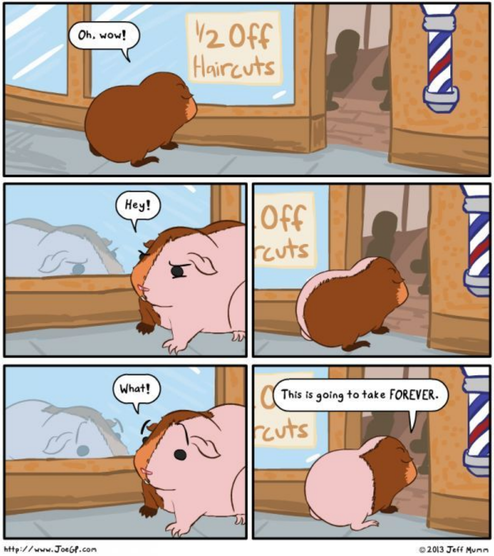
  So look for decreasing and positive terms (eventually) that we can integrate (Calc I or Chap 7) + improper integral. Otherwise the test does NOT help.
- $p-$series $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ conv if $p > 1$ and div if $p \leq 1$ by int test






Oh, wow!
1/2 Off Haircuts
Hey!
Off
rcuts
What!
This is going to take FOREVER.
rcuts
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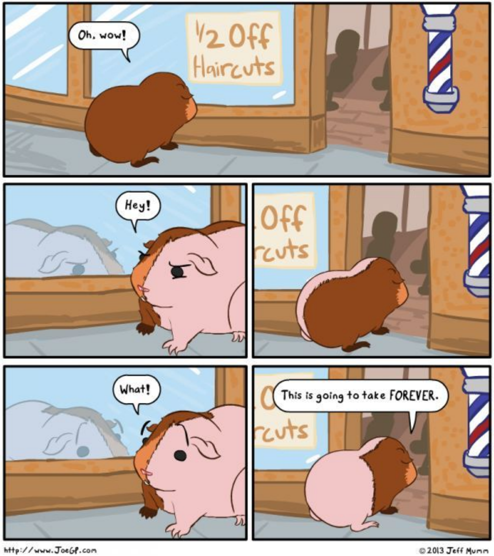

Geo series $\sum_{i=0}^{n-1} \frac{1}{2}\frac{1}{2}^{i} = \sum_{i=1}^{n} \frac{1}{2}^{i}$ converges as $n \to \infty$ to $\frac{\frac{1}{2}}{1-\frac{1}{2}}$ =1 hamster slowly (Zeno's paradox)

Clicker Question

4. Which of the following are true regarding $\sum_{n=2}^{\infty} \frac{2n}{4+n^2}$?

a) It is a geometric series so we can apply 9.2 methods to determine convergence by checking if $|x| < 1$ or divergence otherwise

b) $\lim_{n\to\infty} \frac{2n}{4+n^2} \neq 0$ determines divergence of $\sum_{n=2}^{\infty} \frac{2n}{4+n^2}$

c) We can use linearity to determine convergence

d) We can use the integral test to determine convergence

e) none of the above

Clicker Question

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a) It is a geometric series so we can apply 9.2 methods to determine convergence by checking if $|x| < 1$ or divergence otherwise

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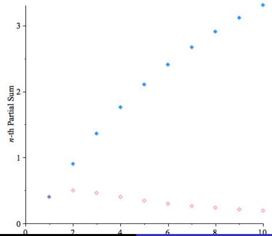

Clicker Question

5. Does the series $\sum_{n=1}^{\infty}(-1)^n$ converge?

a) yes and I have a good reason why
b) yes but I am unsure of why
c) no, but I am unsure of why
d) no, and I have a good reason why
e) it is not a series, so no

Clicker Question

5. Does the series $\sum_{n=1}^{\infty}(-1)^n$ converge?

a) yes and I have a good reason why
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c) no, but I am unsure of why
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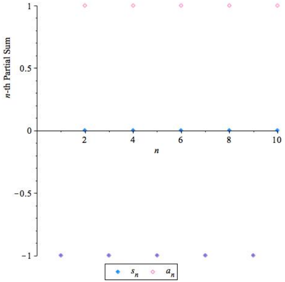

History and Applications

- Brahmagupta gave rules for summing series in his 628 work *Brahmasphutasiddanta (Opening of the Universe)*
- wheat (or rice) and chess problem. Stories of $\sum_{n=0}^{63} 2^n$ grains owed by King (18,446,744,073,709,551,615)
- Nicole Oresme (14th century) harmonic series diverging. Name from wavelengths of the overtones of a vibrating string. Architects.
- James Gregory (1668) introduced the terms *convergence* and *divergence*
- Integral test was developed by Colin Maclaurin and Augustin-Louis Cauchy and is sometimes known as the Maclaurin-Cauchy test (or by either name).
- infinite series are widely used in mathematics & other quantitative disciplines such as physics, computer science, & finance.