

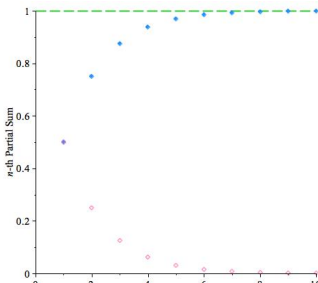
9.2 Geometric Series Review

- Geometric Series a = starting term, x =constant ratio of each term to preceding one

$$\sum_{i=0}^{\infty} ax^i = \frac{a}{1-x} \text{ when } |x| < 1 \text{ and diverges otherwise}$$

n^{th} partial sum (1st n terms added): $\sum_{i=0}^{n-1} ax^i = \frac{a(1-x^n)}{1-x}$ for $x \neq 1$

Example: $\sum_{i=0}^{n-1} \frac{1}{2} \left(\frac{1}{2}\right)^i = \sum_{i=1}^n \frac{1}{2}^i$ careful of starting # and index



9.3 Series: Partial Sums More Generally

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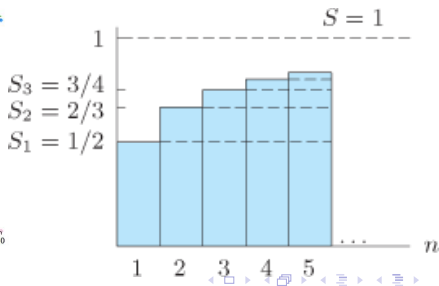
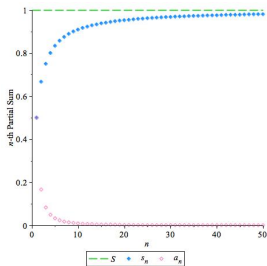
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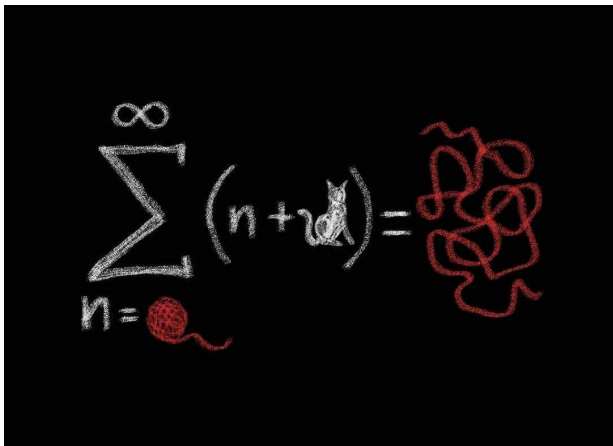
- Example: $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ $S_n = \frac{n}{n+1}$ $\lim_{n \rightarrow \infty} S_n = 1$



9.3: Terms Not Getting Smaller

- terms not getting smaller: $\lim_{n \rightarrow \infty} a_n \neq 0$ or DNE, then partial

sums diverge and so does the series. Example: $\sum_{n=1}^{\infty} \frac{5+n}{2n+1}$

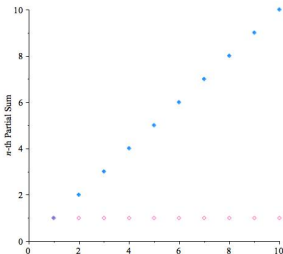


Clicker Question

1. What can we say about the series $\sum_{n=1}^{\infty} (1)^n$?
- a) it is a geometric series with a starting term a and constant ratio of each term to its preceding one x
 - b) we can find a pattern for the partial sums $S_n = \sum_{i=1}^n a_i$
 - c) $\lim_{n \rightarrow \infty} a_n \neq 0$ so we can apply the terms not getting smaller
 - d) all of the above

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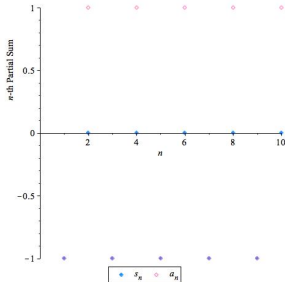
2. Does the series $\sum_{n=1}^{\infty} (-1)^n$ converge?

- a) yes and I have a good reason why
- b) yes but I am unsure of why
- c) no, but I am unsure of why
- d) no, and I have a good reason why
- e) it is not a series, so no

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9.3: Linearity for Convergence or Divergence

- **Linearity:** $\sum_{n=1}^{\infty} a_n$ converges to S and $\sum_{n=1}^{\infty} b_n$ converges to T , and k is any constant, then $\sum_{n=1}^{\infty} ka_n + b_n$ converges to $kS + T$.

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Application 1: add two geometric series (converge to sum)

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Application 1: add two geometric series (converge to sum)

Application 2: add divergent & convergent series (diverge)

Example: $\sum_{n=1}^{\infty} \frac{1}{2}^n + (-1)^n$.

Diverges, because if it were convergent, then subtract convergent $\sum_{n=1}^{\infty} \frac{1}{2}^n$ and the result should converge by linearity, but doesn't!

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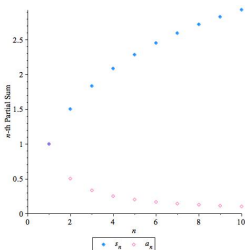
3. Do any of the following apply to $\sum_{n=1}^{\infty} \frac{1}{n}$?

- a) It is a geometric series so we can apply 9.2 methods $\frac{a}{1-x}$ to determine convergence
- b) We can use the $\lim_{n \rightarrow \infty} \frac{1}{n} \neq 0$ to determine divergence of $\sum_{n=1}^{\infty} \frac{1}{n}$
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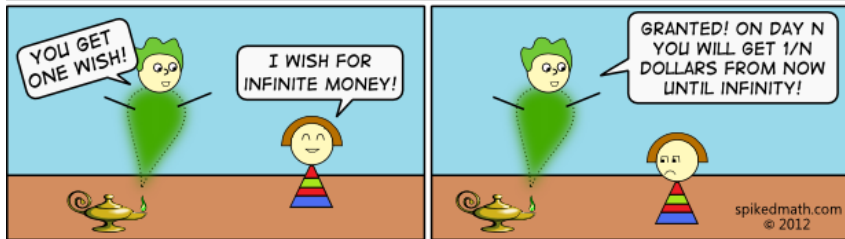


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Geo series $\sum_{i=0}^{n-1} \frac{1}{2} \left(\frac{1}{2}\right)^i = \sum_{i=1}^n \frac{1}{2} \left(\frac{1}{2}\right)^i$ converges as $n \rightarrow \infty$ to $\frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$ hamster slowly (Zeno's paradox)

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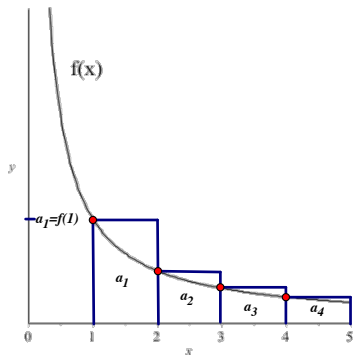


Harmonic series $\sum_{N=1}^{\infty} \frac{1}{N}$ diverges by growing to ∞ slowly! Why?

Integral Test

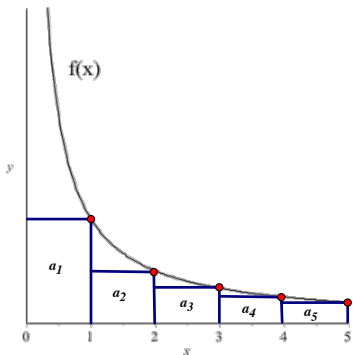
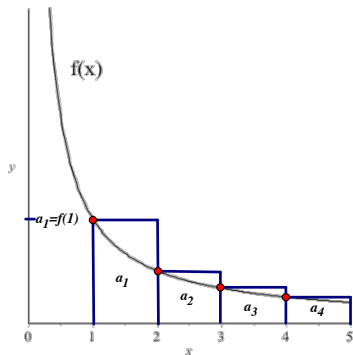
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If series has terms that are decreasing and positive, the integral test not only tells us about convergence, but also bounds the series:



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$$\int_1^{\infty} f(x) dx \leq \sum a_n \leq a_1 + \int_1^{\infty} f(x) dx$$

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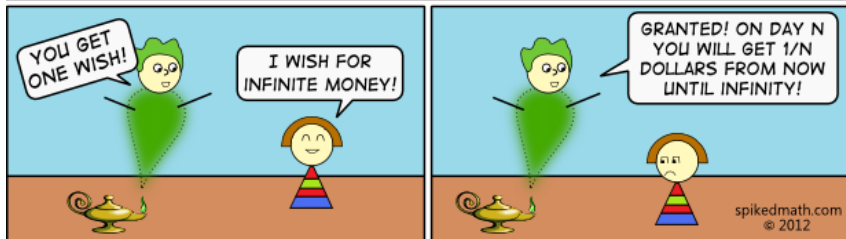


Harmonic series $\sum_{N=1}^{\infty} \frac{1}{N}$ diverges by growing to ∞ slowly! Why?

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$$\int_1^{\infty} \frac{1}{x} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x} dx =$$

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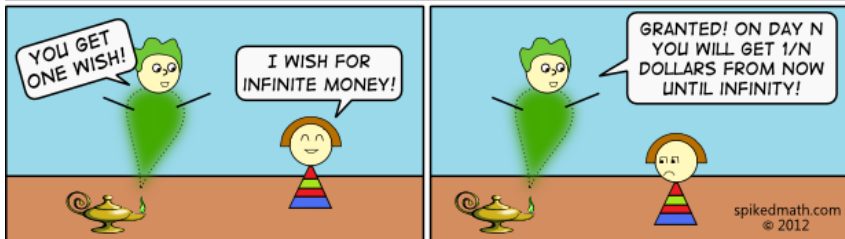


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$$\int_1^{\infty} \frac{1}{x} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x} dx = \lim_{b \rightarrow \infty} \ln(x) \Big|_1^b =$$

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$$\int_1^{\infty} \frac{1}{x} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x} dx = \lim_{b \rightarrow \infty} \ln(x) \Big|_1^b = \lim_{b \rightarrow \infty} \ln(b) - \ln(1)$$

diverges so series does too

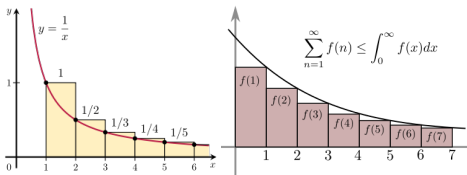
9.3: Integral Test

- For $\sum_{n=1}^{\infty} a_n$, if the terms are decreasing and $a_n > 0$ then the

series behaves the same way as $\int_1^{\infty} a_n dn$.

So look for decreasing and positive terms (eventually) that we can integrate (Calc I or Chap 7) + improper integral. Otherwise the test does NOT help.

- p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ conv if $p > 1$ and div if $p \leq 1$ by int test



Clicker Question

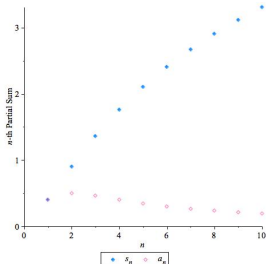
4. Which of the following are true regarding $\sum_{n=2}^{\infty} \frac{2n}{4+n^2}$?

- a) It is a geometric series so we can apply 9.2 methods $\frac{a}{1-x}$ to determine convergence
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History and Applications

- Brahmagupta gave rules for summing series in his 628 work *Brahmasphutasiddhanta (Opening of the Universe)*
- wheat (or rice) and chess problem. Stories of $\sum_{n=0}^{63} 2^n$ grains owed by King (18,446,744,073,709,551,615)
- Nicole Oresme (14th century) harmonic series diverging. Name from wavelengths of the overtones of a vibrating string. Architects.
- James Gregory (1668) introduced the terms *convergence* and *divergence*
- Integral test was developed by Colin Maclaurin and Augustin-Louis Cauchy and is sometimes known as the Maclaurin-Cauchy test (or by either name).
- infinite series are widely used in mathematics & other quantitative disciplines such as physics, computer science, & finance.