Block Matrix Exploration (use either Maple or Matlab):

Goals: Create two "rules"; one for the determinant of a block triangular matrix, and one for the eigenvalues of a block triangular matrix.

1. Recall each:
   (a) The determinant of a triangular matrix is ____________________________.
   (b) The eigenvalues of a triangular matrix is ____________________________.

2. Hypothesis: The determinant of a block triangular matrix is ____________________________.
   Test your hypothesis on the following block matrices \( A = \begin{pmatrix} B & C \\ 0 & D \end{pmatrix} \):
   (a) \( A_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 0 & 9 & 10 \\ 0 & 0 & 11 & 12 \end{pmatrix} \)
      i. What is the determinant of \( A \)?
      ii. What is the determinant of \( B \)?
      iii. What is the determinant of \( D \)?
   (b) \( A_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 0 & 0 & 11 & 12 & 13 \\ 0 & 0 & 14 & 15 & 16 \\ 0 & 0 & 17 & 18 & 19 \end{pmatrix} \)
      i. What is the determinant of \( A \)?
      ii. What is the determinant of \( B \)?
      iii. What is the determinant of \( D \)?

3. Hypothesis: The eigenvalues of a block triangular matrix are ____________________________.
   Test your hypothesis on the block matrices in problem 2.
   (a) Matrix A1:
      i. What are the eigenvalues of \( A \)?
      ii. What are the eigenvalues of \( B \)?
      iii. What are the eigenvalues of \( D \)?
   (b) Matrix A2:
      i. What are the eigenvalues of \( A \)?
      ii. What are the eigenvalues of \( B \)?
      iii. What are the eigenvalues of \( D \)?
Block Matrix Populations: Geographically subdivided populations

Figure 1: Graphs for Geographically subdivided populations.

1. Create the projection matrix, $A$ for the figure ??(b).
2. Find the eigenvalues of $A$ and the eigenvalues of the two diagonal blocks.
3. Which block contains the dominant eigenvalue?
4. Find the eigenvector corresponding to the dominant eigenvalue.
5. What does this say biologically?
6. Change F5 to 1.5 and F6 to 3 and answer question (1)-(5).